Characteristics of Functions

Lesson Synopsis

Students will collect and organize data using various representations. They will identify the characteristics such as independent/dependent variables, domain and range, continuous/discrete, and increasing/decreasing relationships. Students will make predictions from the representations. Students will compare and evaluate functions in y = and f(x) notation. Two extra days are allotted in this lesson in case teachers need time for additional review or want to do supplementary activities.

TEKS:

A.1 Foundations for functions. The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

A.1A Describe independent and dependent quantities in functional relationships.

A.1B Gather and record data and use data sets to determine functional relationships between quantities.

A.1D Represent relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, ~~equations, and inequalities~~.

A.1E Interpret and make decisions, predictions, and critical judgments from functional relationships.

A.2 Foundations for functions. The student uses the properties and attributes of functions.

A.2B Identify mathematical domains and ranges and determine reasonable domain and range values for given situations, both continuous and discrete.

A.2D Collect and organize data, make and interpret scatter plots ~~(including recognizing positive, negative, or no correlation for data approximating linear situations),~~ and model, predict, and make decisions and critical judgments in problem situations.

A.3 Foundations for functions. The student understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations.

A.3A Use symbols to represent unknowns and variables.

A.3B Look for patterns and represent generalizations algebraically.

A.4 Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

A.4A Find specific function values, simplify polynomial expressions, ~~transform and solve equations, and factor as necessary~~ in problem situations.

A.4C Connect equation notation with function notation, such as $y = x + 1$ and $f(x) = x + 1$.

Process TEKS:

8.14 Underlying processes and mathematical tools. The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.

8.14A Identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics.

8.15 Underlying processes and mathematical tools. The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models.

8.15A Communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models.

8.16 Underlying processes and mathematical tools. The student uses logical reasoning to make conjectures and verify conclusions.

8.16B Validate his/her conclusions using mathematical properties and relationships.

GETTING READY FOR INSTRUCTION

Performance Indicator(s):

- Collect and organize a set of data from a problem situation. Represent and describe the data using a table, scatterplot, graph of the function, verbal description, and algebraic generalization. Identify the characteristics of the relation including whether it is a function, independent dependent variables, domain (continuous or discrete) and range, and whether it is increasing or decreasing. Use the representations to make predictions and critical judgments in the problem situation. (A.1A, A.1B, A.1D, A.1E; A.2B, A.2D; A.3A, A.3B)
  ELPS: 3E, 2E, 2I, 3F, 3G, 4F, 5F, 5G
- Simplify and evaluate expressions given in function notation to find specified domain and range values. (A.4A, A.4C)
  ELPS: 3E, 2E, 2I, 3F, 3G, 4F, 5F, 5G

Key Understandings and Guiding Questions:

- Relations can be functions and have specific characteristics such as independent/dependent variables, domain/range, continuous/discrete, and increasing/decreasing.
  - How do you distinguish between independent variables and dependent variables?
  - How can you determine if a relation is a function?
  - What is the domain of a function?
  - How is the domain of a function related to the range of the function?
  - What is the difference between continuous and discrete data?
  - How can you distinguish between increasing and decreasing functions?
- Representations of functions can be used to make predictions and critical judgments in problem situations.
  - What are the various representations of a function?
  - How do the various representations help you make predictions in the problem situation?
- Functional relationships can be represented and evaluated from functional notation ($f(x) = mx + b$).
  - How do $y =$ and $f(x)$ notation compare?
  - If $f(x) = 2x + 3$, what is $f(-2)$?
  - If $f(x) = 4x – 5$, what is the value of x if $f(x) = 3$?

Vocabulary of Instruction:

- relation
- function
- representations
- independent variable
- domain
- range
- continuous
- discrete
- dependent variable
- increasing function
- decreasing function
- f(x) notation

Materials:

- graphing calculator
- chart paper
- chart markers
- sticky dots
- meter stick
- ruler

Resources:

- **STATE RESOURCES:**
  - **Mathematics TEKS Toolkit:** Clarifying Activity/Lesson,/Assessments
    http://www.utdanacenter.org/mathtoolkit/index.php
  - **TEXTEAMS: Algebra 1: 2000 and Beyond:** I – Foundations of Functions; 1. Developing Mathematical Models, 1.1 Variables and Functions, Act 1 (Examples of Dependent Relationships), Act. 2 (Independent and Dependent Variables), 1.2 Valentine's Day Idea; Act. 1 (Valentine's Day Idea), Act. 2 (Using Tables to Find the More Economical Offer), Act. 3 (Using Graphs to Find the Better Idea), Act. 4 (New Rose Offers), Act. 5 (Using Tables for New Rose Offers), Act. 6 (Using Graphs for New Rose Offers); 2. Using Patterns to Identify Relationships, 2.1 Identifying Patterns, Act. 1 (Painting Towers), Act. 2 (Building Chimneys), Act. 3 (Constructing Trucks), Act. 4 (Generating Patterns), Student Act. (Perimeter of Rectangles), 2.2 Identifying More Patterns, Act. 1 (Building Blocks), Act. 2 (Starting Staircases), Act. 3 (Too Many Triangles)

Advance Preparation:

1. Handout: **By the Sea** (1 per student)
2. Handout: **Four Quadrant Grid** (1 per student)
3. Handout: **Miles and Intersections** (1 per student)
4. Transparency: **Miles and Intersections Data Collection** (1 per teacher)
5. Handout: **Frayer Model – Relation** (1 per student)
6. Handout: **Burning Calories** (1 per student)
7. Handout: **Cartesian Coordinate System** (1 per student)
8. Handout (optional): **Independent/Dependent Sentence Strips Connections** (1 per student)
9. Card Set (optional): **Independent/Dependent Sentence Strips** (1 set per group)
10. Handout (optional): **Relations and Dependency** (1 per student)
11. Handout: **Facts About Functions** (1 per student)
12. Handout: **Ticket Prices** (1 per student)
13. Handout: **Order of Operations** (1 per student)
14. Handout: **Order of Operations by Graphing Calculator** (1 per student)
15. Handout: **Function Notation** (1 per student)
16. Handout: **Flying with Functions** (1 per student)
17. Handout: **Take a Look at the Data** (1 per student)
18. Handout: **Analyzing Relations and Functions** (1 per student)

Background Information:

Mathematics, especially algebra, is used to describe and interpret how quantities are related. This lesson examines how one quantity might affect another, and how their relationship is reflected in the graphs and tables that represent them. Students are introduced to the mathematical ideas of relation, dependence, and function. Students build upon their middle school work of creating tables of values, scatterplots, evaluating expressions, and identifying and extending patterns in order to investigate relations and functions.

GETTING READY FOR INSTRUCTION SUPPLEMENTAL PLANNING DOCUMENT

Instructors are encouraged to supplement, differentiate and substitute resources, materials, and activities to address the needs of learners. The Exemplar Lessons are one approach to teaching and reaching the Performance Indicators and Specificity in the Instructional Focus Document for this unit. A Microsoft Word template for this Planning document is located at www.cscope.us/sup_plan_temp.doc. If a supplement is created electronically, users are encouraged to upload the document to their Lesson Plans as a Lesson Plan Resource for future reference.

Instructional Procedures

ENGAGE

1. Distribute the handout: **By the Sea** to each student. Also, be sure each student has their own graphing calculator. Every student needs to learn to enter and analyze data using the graphing calculator.
2. Lead students as a whole group through the set up of the calculator and the entering and graphing of the first set of data points to be entered into L_1 and L_2 of the graphing calculator. Use a view screen or other projection method and make sure each student is following the correct procedures with their graphing calculators.
3. Have students work in pairs to complete the next two sets of data points. Monitor students closely.
4. When students have completed the exercise problems 1-11, discuss the results in whole group.
5. Assign students to work individually on question 12. This may be completed as homework, if necessary.

Notes for Teacher

NOTE: 1 day = 50 minutes
Suggested Day 1
MATERIALS

- Handout: **By the Sea** (1 per student)
- Handout: **Four Quadrant Grid** (1 per student)
- graphing calculators

TEACHER NOTE
Students will enter data into the statistical list feature of the graphing calculator, and graph the line plot using the statistical plot feature.

TEACHER NOTE
Students plotted points in middle school. Some students may have used the graphing calculator, while for others it will be completely new. Being able to use the graphing calculator to analyze data is an extremely important tool in

Instructional Procedures

EXPLORE/EXPLAIN 1

1. Have volunteer students post their Four Quadrant Grids with their diagram and point sets. Some students may actually enter their data into a display calculator to show to the class.
2. Engage students in a short discussion about how they get to school each day.
   - **How many walk to school?**
   - **How many ride the bus to school?**
   - **How many are driven to school?**
   - **How many depend on someone else to get to school?**
   - **How many miles do you travel to get to school?**
   - **How many intersections do you go through on the way to school?**
3. Distribute the handout: **Miles and Intersections** to each student. Read the introductory paragraph on the handout.
4. Ask:
   - **Do you think there is a relationship between the distance from school and the number of intersections you must go through?** Accept any answer at this point. Students might think that the longer the distance traveled the more intersections they will encounter.
5. Put students in groups of two or three. Go over #1a & #1b in whole group discussion. Give students time to estimate their intersections and distances and fill in the values on #1a & #1b.
6. Collect students' responses on the board using the transparency: **Miles and Intersections Data Collection**. Have students copy the data on the transparency table to the table on their handout: **Miles and Intersections**, question #1c.
7. Have students work in their groups to answer #1d. Share answers to question #1d in whole group.
8. Discuss with students appropriate scales and labels for the axes of the graph in #2. Give students time to plot the points by hand and then enter them into the graphing calculator to create a scatterplot.
9. Have students work in their groups to complete the handout: **Miles and Intersections**, questions #3-9.
10. Conduct a whole class discussion. Have students explain and compare their responses.

Notes for Teacher

Algebra 1.

TEACHER NOTE
If students are having trouble deciding what diagram to draw on question 12, suggest using their three initials done in block letters. If a student does not have three initials, have them pick another letter.

TEACHER NOTE
Give students a **heads-up** that they will need to know the miles they travel to school and the number of intersections they cross for the activity for the next class period.

Suggested Day 2

MATERIALS
- Handout: **Miles and Intersections** (1 per student)
- Transparency: **Miles and Intersections Data Collection** (1 per teacher)
- Handout: **Frayer Model – Relation** (1 per student)
- graphing calculator

TEACHER NOTE
Mathematics, especially algebra, is used to describe and interpret how quantities are related. In this activity students will explore how one quantity might affect another, and how their relationship is reflected in the graphs and tables that represent them.

TEACHER NOTE
Students will collect data on the number of intersections and the number of miles from their home to school.

TEACHER NOTE
Students may need help in estimating mileage and number of intersections. If possible, use MapQuest or other online map resource to make maps of the area serviced by the school for reference.

TEACHER NOTE
When collecting student data the teacher may need to add ordered pairs:
1. One ordered pair that matches a

Instructional Procedures

- **Is there a definite pattern in the table or the scatterplot?** *Elicit the idea that there is no strong pattern in the table or scatterplot. This will be become more apparent as they continue the lesson.*
- **Although there is no pattern in the table or scatterplot that represents the number of intersections, the distance and the number of intersections are *related.* How does the table and scatterplot show a relation between the distance and the number of intersections?** Question 6 hints at the mathematical idea of **relation**, but students will probably struggle with this question. Accept answers, and probe with questions such as:
- **What is common to both the table and scatterplot?**
- **How did the table help you make the scatterplot?**

11. As soon as all students understand that the table and scatterplot are composed of ordered pairs, introduce the formal definition of **relation.** A **relation** between two quantities is a set of ordered pairs of the form (*x, y*). Fill in and answer question 10 in whole group.
12. Ask questions for understanding:
    - **What are the two quantities in this activity?** *(distance, number of intersections)*
    - **Give one of the ordered pairs in the relation.** *Answers may vary. Sample answer is (3 miles, 4 intersections)*
13. Distribute the handout: **Frayer Model – Relation** to each student.
14. Have students complete the Frayer Model to define and give examples of a relation. This can be completed as homework, if necessary.

Notes for Teacher

different number of intersections to a distance already given by a student (ensuring the data does not represent a function), and

2. One ordered pair that either:
   a) matches a large number of intersections with a short distance, or
   b) matches a small number of intersections with a large distance.

The purpose of adding the data points is to manipulate the data to be a non-functional relationship. This will provide counter examples as they begin to examine the meaning of function.

INFORMAL OBSERVATION
As you walk around the room, observe how students are creating their scatterplot. Is the distance the x value (independent) and the number of intersections (dependent)? If there is confusion, redirect with the following questions.

- **Which axis on the graph is the x-axis?**
- **Are we graphing distance or number of intersections on the x-axis?**

TEACHER NOTE
Make sure when students give an example of the ordered pair, they use labels to indicate what each value represents.

VOCABULARY NOTE
The Frayer Model for the definition of **relation** is one of the best practice strategies for all students, especially ELLs.

TEACHER NOTE
Remind students to keep all their work throughout this lesson. They will refer to it multiple times in this and future lessons.

TEACHER NOTE
Keep one of the sets of data to post later in Lesson 2 when students compare all the functions studied.

EXPLORE 2

1. Debrief student responses on handout: **Frayer Model – Relation** by having them share their models with the class.

Suggested Day 3

MATERIALS
- Handout: **Burning Calories** (1 per

Instructional Procedures

2. Put students in pairs or small groups.
3. Distribute the handout: **Burning Calories** to each student.
4. Have student complete questions 1-4 in the activity. Have each group member select a different activity and compare the tables and graphs of the different activities. Have the group select one activity and create a display on chart paper. Include the table and graph on the display.
5. As a whole group discuss results on the displays.
6. Have students complete the rest of the questions individually. These questions can be completed as homework, if necessary.

Notes for Teacher

student)

- graphing calculator
- chart paper
- chart markers

TEACHER NOTE
In this activity students are introduced to the mathematical idea of dependence. Students will be investigating the relationship between the number of minutes (y) it takes to burn off (x) Calories. The inverse relationship between the Calories burned (y) and the number of minutes (x) is another way to compare these variables. However, the relationship between the number of minutes (y) it takes to burn off (x) Calories is an important relationship to dieters who know the amount of calories they have taken in and want to determine the number of minutes of exercise they need to complete. This is an example of the fact that time is not always the independent variable.

TEACHER NOTE
Students can graph more than one activity, but it is important that only one function is on a graph. When defining functions later in the lesson, it is difficult for students to understand the vertical line test if more than one graph is on the plane.

TEACHER NOTE
Suggest that students extend their graphs beyond 800 kcal so that they can extend for question 6.

TEACHER NOTE
Remind students this handout will be used later in the lesson for another activity. Do not lose it.

TEACHER NOTE
Keep at least one of the better posters to post later in Lesson 2 when students compare all the functions studied.

EXPLAIN 2

Day 4

1. Debrief **Burning Calories** by having students share responses to questions.
2. Distribute the handout: **Cartesian Coordinate System** to each student.
3. Go over p. 1 and the top of p. 2 in whole group, while students fill in their

Suggested Days 4-5
MATERIALS

- Handout: **Cartesian Coordinate System** (1 per student)
- Handout: **Facts about Functions** (1 per student)

Instructional Procedures

worksheets.

4. Have students work in pairs to complete the Connections questions.
5. Have students share out results in whole group discussion of the questions.

If students need additional practice on independent and dependent relations, **Independent/Dependent Sentence Strips** may be used to reinforce these concepts. The handout: **Relations and Dependency** can be used as additional practice or homework.

Day 5

1. Distribute the handout: **Facts about Functions** to each student.
2. Go over example problems #1-5. The first problem of each type can be worked as an example and the second problem as guided practice. Students should have covered mapping in middle school but still may need a review. For most students use and symbolism of open and closed dots on example #4 will be a new concept. Extra explanation may be required.

$<$ ○ $\le$ ●

D: $\{-1 \le x < 2\}$ or $\{x \mid -1 \le x < 2\}$
R: $\{-4 \le y < 5\}$ or $\{y \mid -4 \le y < 5\}$

3. Have students work individually on Practice Problems. These may be completed as homework, if necessary.

Notes for Teacher

TEACHER NOTE
The focus of this activity is to identify characteristics of relations and functions. Students will also compare and contrast the relationships in **Miles and Intersections** and **Burning Calories**.

TEACHER NOTE
Although students worked with real numbers in middle school, they may not be comfortable with the definition of “real.” You may need to give students some examples of rational numbers, i.e., 5, ½, -3, 2.5, 100, -¼. Explain that rational numbers are any number that can be written as a fraction.

TEACHER NOTE
Because domain and range may be represented in various ways on state and standardized tests, introduce students to a variety of methods during instruction. Methods found on various testing situations are used in this lesson.

TEACHER NOTE
Students will need to refer to their work in the handout **Miles and Intersections** and **Burning Calories**

SUPPLEMENTAL MATERIALS

- Handout: **Independent/Dependent Sentence Strips Connections** (1 per student)
- Cards: **Independent/Dependent Sentence Strips** (1 set per group)
- Handout: **Relations and Dependency** (1 per student)

If students need further clarification of independent and dependent relationships, the supplemental activity **Independent/Dependent Sentence Strips** can be used. **Relations and Dependency** is a good review of relations, dependency, and independent/dependent.

STATE RESOURCES
TEXTEAMS: Algebra 1: 2000 and Beyond
I – Foundations of Functions; 1. Developing Mathematical Models, 1.1 Variables and Functions, Act 1

Instructional Procedures

ELABORATE 1

1. Distribute the handout: **Ticket Prices** to each student.
2. Have a student read the first paragraph aloud. Discuss the questions with students in whole group.
3. Have another student read the problem aloud.
4. Put students in groups to complete questions 1 and 2. Have each group collect a sheet of chart paper and chart markers. Have each group make a display of the table and scatterplot that represent the relation. Post display charts to use when debriefing questions.
5. Have students complete questions 3-12 in pairs or small groups. Monitor students and their conversations to determine their understanding of the characteristics of relations and functions.
6. Share out results in whole group discussion using display charts to verify answers.
7. Assign questions 13-16. Students will need to use previous handouts (**Miles and Intersections**, **Burning Calories**, and **Ticket Prices**) to answer the questions. If necessary, students can complete these as homework.

Notes for Teacher

(Examples of Dependent Relationships), Act. 2 (Independent and Dependent Variables) may be used to reinforce these concepts or used as alternate activities.

Suggested Day 6

MATERIALS:

- Handout: **Ticket Prices** (1 per student)
- graphing calculator
- chart paper
- chart markers

TEACHER NOTE

In this activity students will be given a symbolic representation of a functional relationship. They will represent the relationship using various models and use the models to develop the concepts of dependency and functions.

INFORMAL OBSERVATION

Monitor students and help students to answer problems by asking probing questions.

- **What must you do to fill in the y values in the table?**
- **How can you determine the independent and dependent values?**
- **How can you determine if a relation is a function?**
- **How can you distinguish between discrete and continuous data?**
- **How can you tell from a graph if the relation is increasing or decreasing?**

MISCONCEPTION

Students may mistake variables as letters representing an object as opposed to representing the number or quantity of objects.

STATE RESOURCES

TEXTEAMS: Algebra 1: 2000 and Beyond

I – Foundations of Functions; 1. Developing Mathematical Models, 1.2 Valentine's Day Idea; Act. 1 (Valentine's Day Idea), Act. 2 (Using Tables to Find the More Economical

Instructional Procedures

Notes for Teacher

Offer), Act. 3 (Using Graphs to Find the Better Idea), Act. 4 (New Rose Offers), Act. 5 (Using Tables for New Rose Offers), Act. 6 (Using Graphs for New Rose Offers) may be used to reinforce these concepts or used as alternate activities.

TEACHER NOTE
Keep at least one of the better posters to post later in Lesson 2 when students compare all the functions studied.

EXPLORE/EXPLAIN 3

Day 7

1. Debrief the previous activity by putting the definition of *function* on the board. Have students determine if they want to change any of their answers to #13-16 on handout: **Ticket Prices**.

    A **function** is a dependence of one quantity on another in which *exactly* one value of the range (the dependent variable) is paired with one value of the domain (the independent variable).
    A function is always a relation, but a relation is not always a function.

Order of operations is a student expectation from Grade 7. It is reviewed in Grade 8 with rational numbers. If students still need additional practice on order of operations, extra time can be spent on **Order of Operations** and **Order of Operations by Graphing Calculator** before beginning **Function Notation**.

2. Distribute the handout: **Order of Operations** to each student. Students should not use calculators for this activity.
3. Go over the steps for order of operations at the top of p. 1.
4. Have students work in small groups on problems #1-6. Instruct them to work the problem independently, then compare answers with others in the group. When all group members have finished the six problems, share out correct answers and order of operation steps for each problem in whole group discussion.
5. Go over problems #7-9 on p. 2 in whole group instruction.
6. Distribute calculators to each student.
7. Distribute the handout: **Order of Operations by Graphing Calculator** to each student.
8. Instruct students to find the answer to #1 without using a calculator. Then have students use the calculator to find the answer for #1. Ask students if their answers were the same. Most students will not know to put parentheses around the -4. Explain to students that the calculator is actually following the order of operations. Raising the 4 to a power would come before multiplying by the negative. The -4 must be put in parentheses in order for the calculator to also square the negative.
9. Work #7 in whole group to explain how to enter numerators and denominators with parentheses. Work #13 in whole group to explain how to use multiple parentheses and not enter brackets when using the graphing calculators. Brackets are used when dealing with matrices.
10. Have students work in pairs to complete the remainder of the problems on the handout: **Order of Operations by Graphing Calculator**. Remind

Suggested Days 7-8

MATERIALS

- Definition of function and sentence relating functions and relations written on the board or on a transparency to use in class discussion.
- Handout: **Order of Operations** (1 per student)
- Handout: **Order of Operations by Graphing Calculator** (1 per student)
- Handout: **Function Notation** (1 per student)
- Handout: **Flying with Functions** (1 per student)
- graphing calculator

TEACHER NOTE
Order of operations is a student expectation from Grade 7. It is reviewed in Grade 8 with rational numbers. The first activity will be a review of order of operations and should be completed without calculators. The next activity introduces students to order of operations on the graphing calculator. Without and with calculators is done in one day. If necessary, this can be extended to two days.

Instructional Procedures

students that parentheses must be used for all inclusion symbols, and numerators and denominators must be put in parentheses. Instruct students that each student should find the answer using the calculator on their own and then check the result with their partner. This should be finished in class so that students have access to the graphing calculators.

11. Have students work the Practice Problems on the handout: **Order of Operations** independently. If necessary, this may be completed as homework.

Day 8

1. Debrief the homework by having students check the Practice Problems on the handout: **Order of Operations** using the calculators. .
2. Distribute the handout: **Function Notation** to each student.
3. Go over the notes and examples in whole group. Students should not use graphing calculators during this section of the activity.
4. Have students work the practice problems 1-4 independently. When finished have students pair up to check answers. Go over any concept misconceptions in whole group.
5. Have students continue to work on questions 5-10 in pairs. Have pairs share answers in whole group discussion.
6. Students will now begin using the graphing calculator. Go over the notes using the graphing calculator to find function values on p. 3.
7. Have students use the graphing calculator to verify and justify their results on questions 1-10.
8. Distribute the handout: **Flying with Functions** to each student. Have students work independently on the activity. This may be completed as homework if necessary.

Notes for Teacher

TEACHER NOTE
In the last activity students will be introduced to function notation and connect function notation, f(x), to y = notation. Students will also find specific function values.

TEACHER NOTE
The graphing calculator can be used to help students understand the concept of function notation. It should not be used prior to student practice with manual substitution. With the calculator the students can enter the function in y_1, then on the home screen enter $y_1(2)$, and it will give the solution for f(2). These directions are included on the student handout. Multiple functions can be entered using y_1, y_2, y_3, and so on. Students could also use the table to determine function values. The benefit of the home screen entry method is that it mimics the use of f(x) notation. Using the table in the calculator does not emphasize f(x) notation.

ELABORATE 2

1. Allow students some time to verify answers on the handout: **Flying with Functions** using the graphing calculators. Debrief the homework using the following questions:
   - **What does the 5 represent in f(5)?**
   - **What does the final answer represent in the symbolic form?**
   - **Why is function notation helpful in working these problems?**
   - **How can you use the graphing calculator to verify your results?**
2. Distribute the handout: **Take a Look at the Data** to each student.
3. Students may complete the activity in pairs or small groups. Monitor student carefully to check for understanding.
4. This activity may be completed for homework, if necessary.

Suggested Day 9
MATERIALS

- Handout: **Take a Look at the Data** (1 per student)
- graphing calculator

TEACHER NOTE
Students will investigate and analyze data that represents relations. Some is functional and some is not. Linear and quadratic functions will be represented.

STATE RESOURCES
TEXTEAMS: Algebra 1: 2000 and Beyond
I – Foundations of Functions; 2. Using Patterns to Identify Relationships, 2.1 Identifying Patterns, Act. 1 (Painting Towers), Act. 2 (Building Chimneys), Act. 3 (Constructing Trucks), Act. 4

Instructional Procedures	Notes for Teacher
	(Generating Patterns), Student Act. (Perimeter of Rectangles) may be used to reinforce these concepts or used as alternate activities.

EVALUATE

1. Debrief **Take a Look at the Data** in whole group as a review.
2. Distribute the handout: **Analyzing Relations and Functions** to each student.
3. Students should work independently to complete the activity.

Suggested Day 10

MATERIALS:

- Handout: **Analyzing Relations and Functions** (1 per student)
- graphing calculator

TEACHER NOTE

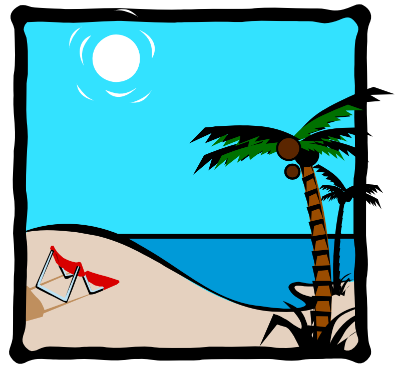
This activity is to assess student understanding of characteristics of relations and functions. Students should do their work individually and not in groups.

TAKS CONNECTION

Grade 9 TAKS 2003 #24, 25, 42, 46, 49
Grade 10 TAKS 2003 #14, 19, 37, 43, 45
Grade 11 TAKS 2003 #28, 45, 54, 59, 60
Grade 9 TAKS 2004 #24, 46, 50
Grade 10 TAKS 2004 #4, 12, 23, 35, 51
Grade 11 TAKS 2004 #27, 31, 55

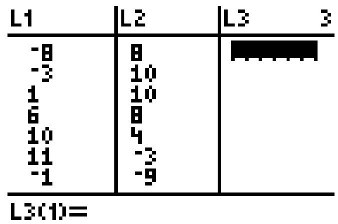
Grade 9 TAKS 2006 #25, 33, 41
Grade 10 TAKS 2006 #10, 14, 38, 48
Grade 11 TAKS 2006 #12, 13, 17, 40, 48, 54, 59

By the Sea (pp. 1 of 2) KEY



The graphing calculator can be used to plot points. Points are entered under the statistics function (STAT) of the calculator. Points are then plotted using the STAT PLOT. These skills will be used over and over in future concepts when entering data and finding functions.

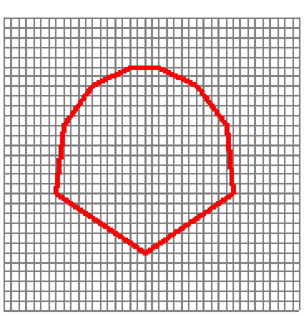
1. Turn off the axes to the graphing window. This is found under format.
2. Set the WINDOW values as X(-20, 20, 1) and Y(-15, 15, 1).
3. Under the STAT/EDIT keys, enter the following points in L1 (x value) and L2 (y value). Be sure to enter them in the order given, left to right.
   (-8, 8) (-3,10) (1, 10) (6, 8) (10, 4) (11, -3) (-1, -9) (-13, -3) (-12, 4) (-8, 8) Your lists will look like the picture below for these ordered pairs. You will need to scroll down to see the last three pairs on the screen. Use this same model for entering your lists in #6 and #9 below.

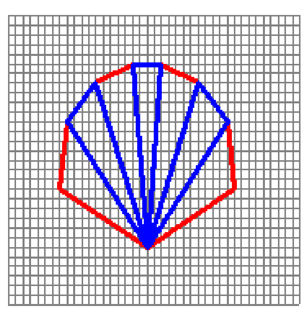


4. Go to STAT PLOT and turn on the first plot. Under Type use the second graph. This draws and connects the points. Under Xlist put L1. Under Ylist put L2. Under Mark use the last small point.
5. Press GRAPH. What do you see? What are you making? Draw a quick sketch of the present graph.
   Answers will vary. It should resemble the outline of a sea shell. See below

6. Under the STAT/EDIT keys, enter the following points in L3 (*x* value) and L4 (*y* value). Be sure to enter them in the order given, left to right.
   (-1, -9) (10, 4) (6, 8) (-1, -9) (1, 10) (-3, 10) (-1, -9) (-8, 8) (-12, 4) (-1, -9)
7. Go to STAT PLOT and turn on the second plot. Under Type use the second graph. This draws and connects the points. Under Xlist put L3. Under Ylist put L4. Under Mark use the last small point.
8. Press GRAPH. What do you see? Are you getting a better idea yet? Draw a quick sketch of the present graph.

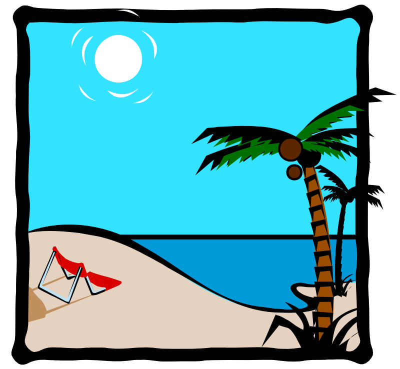
(5)





(9)

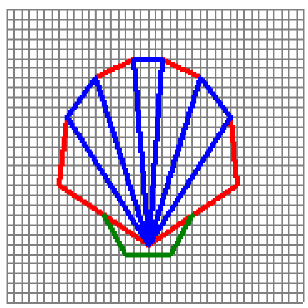
By the Sea (pp. 2 of 2) KEY



9. Under the STAT/EDIT keys, enter the following points in L5 (x value) and L6 (y value). Be sure to enter them in the order given, left to right.
(5, -6) (2, -10) (-4, -10) (-7, -6)

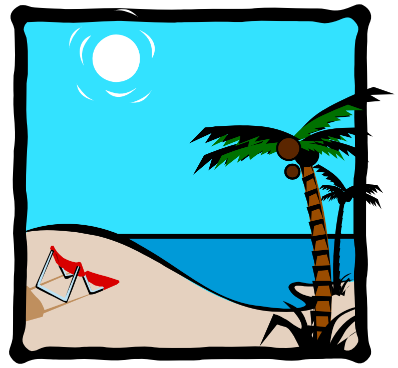
10. Go to STAT PLOT and turn on the third plot. Under Type use the second graph. This draws and connects the points. Under Xlist put L5. Under Ylist put L6. Under Mark use the last small point.

11. Press GRAPH. Doesn't that make you wish you were at the beach right now? Draw a sketch of the final graph.



12. Develop a design of your own. Remember, points must connect in order! You must use all six lists and three scatter plots. Sketch your design on grid paper, labeling all points. Test and verify results using the graphing calculator.

By the Sea (pp. 1 of 2)

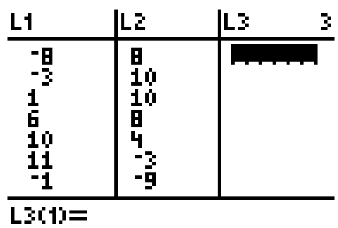


The graphing calculator can be used to plot points. Points are entered under the statistics function (STAT) of the calculator. Points are then plotted using the STAT PLOT. These skills will be used over and over in future concepts when entering data and finding functions.

1. Turn off the axes to the graphing window. This is found under format.

2. Set the WINDOW values as X(-20, 20, 1) and Y(-15, 15, 1).

3. Under the STAT/EDIT keys, enter the following points in L1 (x value) and L2 (y value). Be sure to enter them in the order given, left to right.
   (-8, 8) (-3,10) (1, 10) (6, 8) (10, 4) (11, -3) (-1, -9) (-13, -3) (-12, 4) (-8, 8) Your lists will look like the picture below for these ordered pairs. You will need to scroll down to see the last three pairs on the screen. Use this same model for entering your lists in #6 and #9 below.



4. Go to STAT PLOT and turn on the first plot. Under Type use the second graph. This draws and connects the points. Under Xlist put L1. Under Ylist put L2. Under Mark use the last small point.

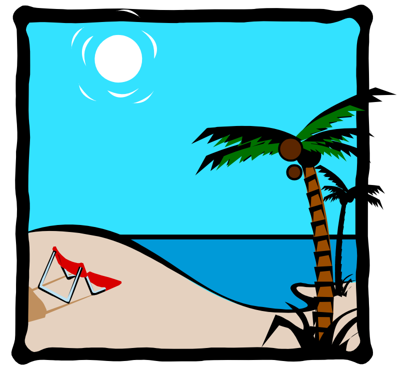
5. Press GRAPH. What do you see? What are you making? Draw a quick sketch of the present graph.

6. Under the STAT/EDIT keys, enter the following points in L3 (*x* value) and L4 (*y* value). Be sure to enter them in the order given, left to right.
   (-1, -9) (10, 4) (6, 8) (-1, -9) (1, 10) (-3, 10) (-1, -9) (-8, 8) (-12, 4) (-1, -9)

7. Go to STAT PLOT and turn on the second plot. Under Type use the second graph. This draws and connects the points. Under Xlist put L3. Under Ylist put L4. Under Mark use the last small point.

8. Press GRAPH. What do you see? Are you getting a better idea yet? Draw a quick sketch of the present graph.

By the Sea (pp. 2 of 2)



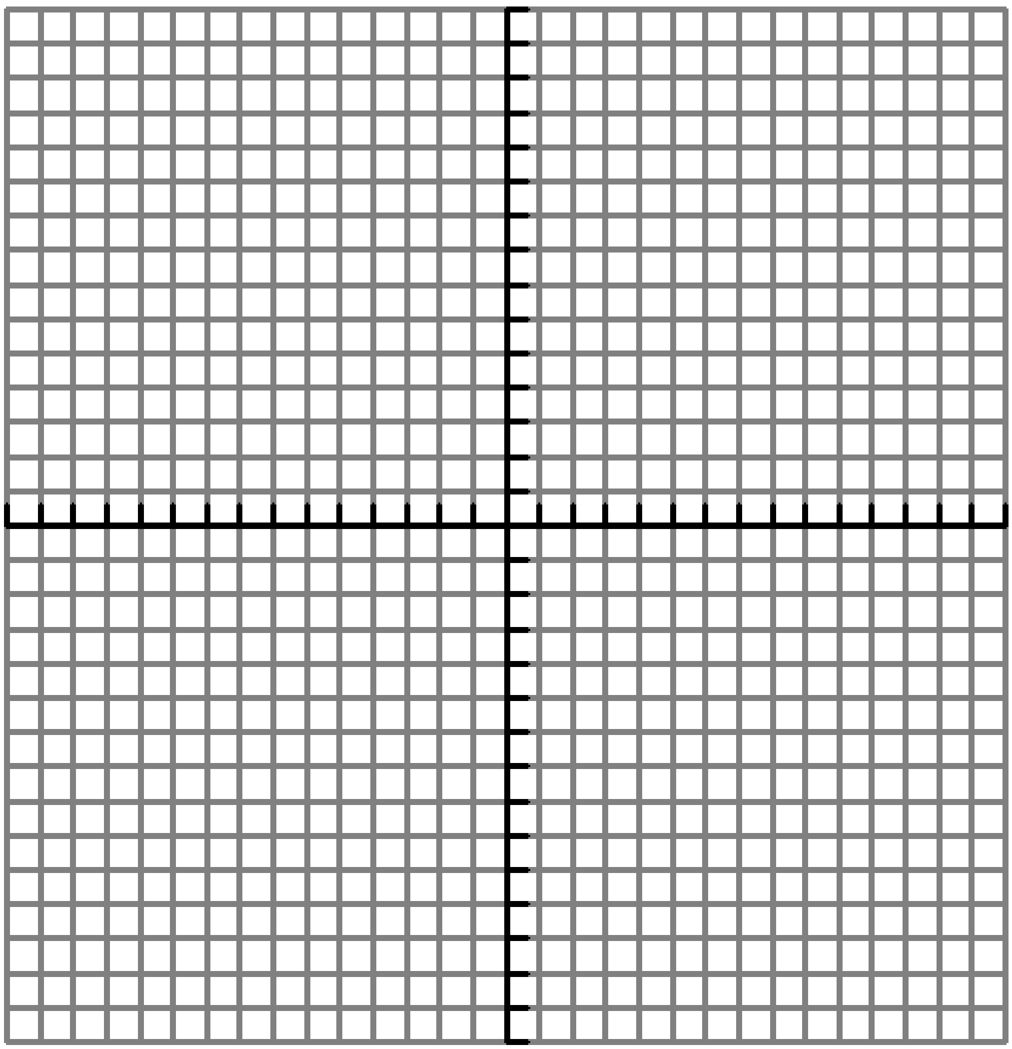
9. Under the STAT/EDIT keys, enter the following points in L5 (x value) and L6 (y value). Be sure to enter them in the order given, left to right.
(5, -6) (2, -10) (-4, -10) (-7, -6)

10. Go to STAT PLOT and turn on the third plot. Under Type use the second graph. This draws and connects the points. Under Xlist put L5. Under Ylist put L6. Under Mark use the last small point.

11. Press GRAPH. Doesn't that make you wish you were at the beach right now? Draw a sketch of the final graph.

12. Develop a design of your own. Remember, points must connect in order! You must use all six lists and three scatter plots. Sketch your design on grid paper, labeling all points. Test and verify results using the graphing calculator.

Four Quadrant Grid



Set A Points:

Set B Points:

Set C Points:

Miles and Intersections (pp. 1 of 3) KEY

Mathematics, especially algebra, is used to describe and interpret how quantities are related. In this lesson you will explore how one quantity might affect another, and how their relationship is reflected in the graphs and tables that represent them.

How many intersections are there on your route from home to school? Is there a relationship between the distance in miles from school and the number of intersections?

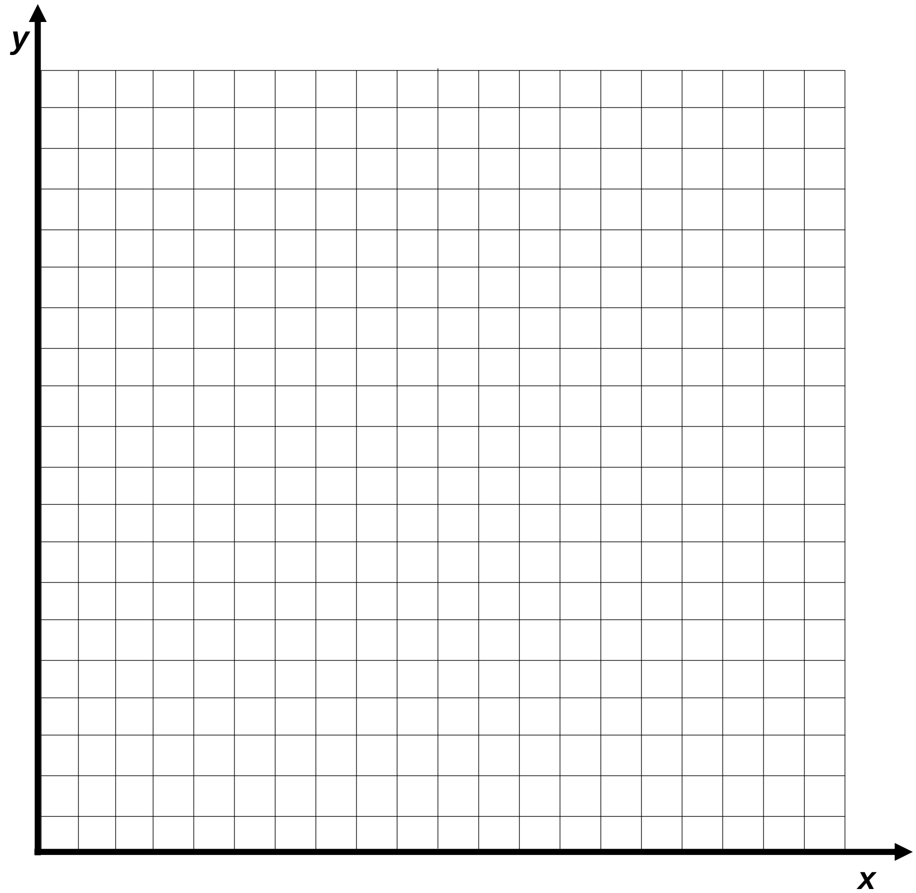
1. Think about the route from your home to school.
   a. Estimate the distance in miles from home to school. Round your estimate to the nearest half-mile.
   Answers will vary.
   b. Write the number of intersections you go through.
   Answers will vary.
   c. Collect the data from your class and enter it in the table below.
   Collect data and display on *Miles and Intersections Data Collection* transparency

Distance (miles)	**Number of Intersections**	**Distance (miles)**	**Number of Intersections**

   d. Describe any patterns you observe in the table. If there is no pattern, say so.
   There should be no strong pattern.

Miles and Intersections (pp. 2 of 3) KEY

2. Create a scatterplot of the class data. Graph the distance along the *x*-axis and the number of intersections along the *y*-axis.



3. Describe the scatterplot. What patterns do you observe? If there is no pattern, say so.
   There should be no strong pattern, or points may be clustered fairly tightly together. The pattern may be slightly linear or increasing.

4. Predict the number of intersections there will be if you live 7 miles from school. Explain how you made your prediction.

   Answers will vary.

Miles and Intersections (pp. 3 of 3) KEY

5. How does the number of intersections change as the distance from home to school increases?
   Students might say the number of intersections increases as the distance increases; however, there should be no definitive answer.

6. Can the distance traveled be used to reliably predict the number of intersections? Why or why not?
   No, the distance traveled cannot be used to reliably help predict the number of intersections. (It is possible that the scatterplot will show a weak positive trend.)

7. How do the table and scatterplot support your answer to question 6?
   There are no strong patterns in either the table or scatterplot.

8. Compare your prediction from question 4 with the rest of the class. Were your predictions the same or different?
   Predictions should be different.

9. How does your answer to question 6 help explain your answer to question 8 above?
   Since there is no pattern, there is no way to use the distance to reliably predict the number of intersections. As a result students will have different answers.

10. Although there is no pattern in the table or scatterplot that represents the number of intersections, the distance and the number of intersections are **related.** How does the table and scatterplot show a **relation** between the distance and the number of intersections?
    Answers will vary. You can plot a set of points which is a relation.

Keep your work from this exploration!

Miles and Intersections (pp. 1 of 3)

Mathematics, especially algebra, is used to describe and interpret how quantities are related. In this lesson you will explore how one quantity might affect another, and how their relationship is reflected in the graphs and tables that represent them.

How many intersections are there on your route from home to school? Is there a relationship between the distance in miles from school and the number of intersections?

1. Think about the route from your home to school.
   a. Estimate the distance in miles from home to school. Round your estimate to the nearest half-mile.

   b. Write the number of intersections you go through.

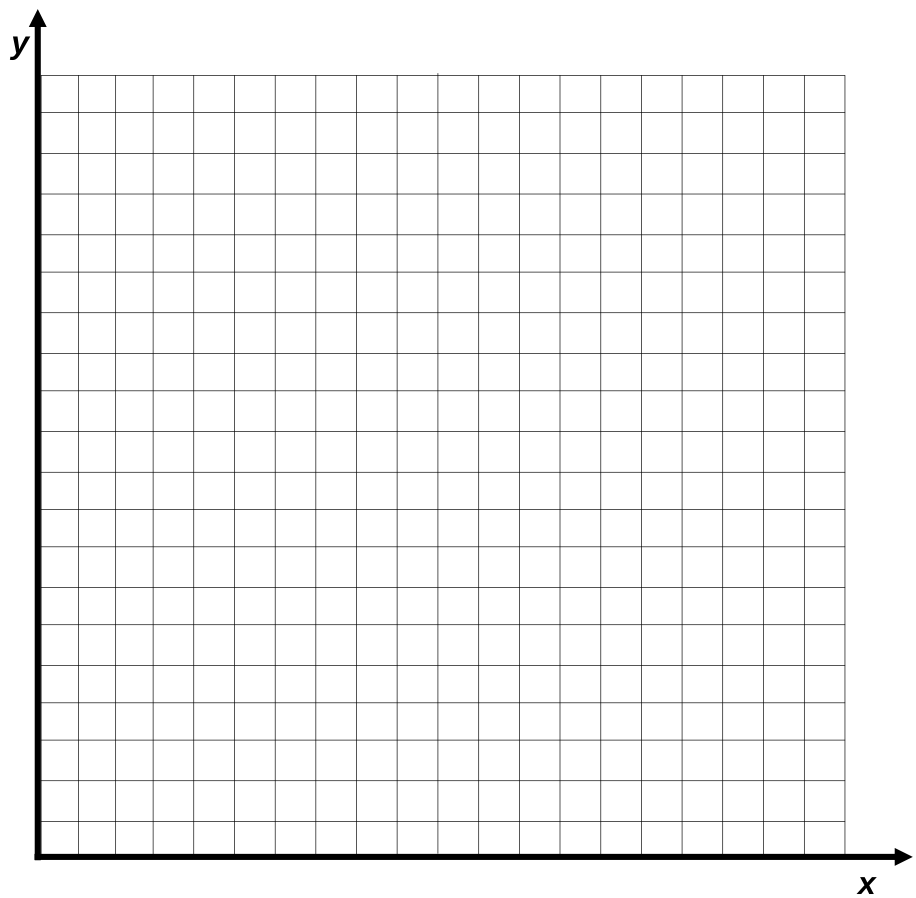
   c. Collect the data from your class and enter it in the table below.

Distance (miles)	Number of Intersections	Distance (miles)	Number of Intersections

   d. Describe any patterns you observe in the table. If there is no pattern, say so.

Miles and Intersections **(pp. 2 of 3)**

2. Create a scatterplot of the class data. Graph the distance along the *x*-axis and the number of intersections along the *y*-axis.



3. Describe the scatterplot. What patterns do you observe? If there is no pattern, say so.

4. Predict the number of intersections there will be if you live 7 miles from school. Explain how you made your prediction.

Miles and Intersections (pp. 3 of 3)

5. How does the number of intersections change as the distance from home to school increases?

6. Can the distance traveled be used to reliably predict the number of intersections? Why or why not?

7. How do the table and scatterplot support your answer to question 6?

8. Compare your prediction from question 4 with the rest of the class. Were your predictions the same or different?

9. How does your answer to question 6 help explain your answer to question 8 above?

10. Although there is no pattern in the table or scatterplot that represents the number of intersections, the distance and the number of intersections are ________________**.** How does the table and scatterplot show a ________________ between the distance and the number of intersections?

Keep your work from this exploration!

Miles and Intersections Data Collection KEY

Students should give their best estimates. This should be a relation that is not a function, although it may show a slightly increasing tendency. The answers will vary depending on student data.

Distance (miles)	Number of Intersections	Distance (miles)	Number of Intersections

Miles and Intersections Data Collection

Distance (miles)	Number of Intersections	Distance (miles)	Number of Intersections

Frayer Model—Relation

Definition (In own words)	**Characteristics**
Examples (from own life)	**Non-examples** (from own life)

Word:

Relation

Burning Calories (p. 1 of 3) KEY

In **Miles and Intersections**, you learned that two quantities such as (*distance, number of intersections*) is a **relation** as long as it forms a set of ordered pairs**.** You also learned that when there is no pattern in the data, it is difficult to make predictions. What if there is a pattern? What if changing one quantity changes the other quantity in a predictable way? In this exploration, you will investigate another relation with special characteristics.

The food you eat provides your body the energy it needs to maintain body functions such as temperature regulation, blood circulation, bone growth, and muscle repair. It also provides the energy needed to be physically active. When you engage in physical activity, you burn ***kilocalories***. A kilocalorie (kcal) is the amount of energy required to raise the temperature of one liter (1 L) of water one degree Celsius (1° C). A dietary Calorie (with a capital *C*) is equal to one kilocalorie.

1. Imagine you are going to the gym after school. You have had extra snacks during the day. Does the number of minutes you need to exercise depend on the number of kilocalories you want to work off? Explain why or why not.
   Answers will vary. Sample: Yes, the longer I exercise the more kilocalories I burn.

   The table below shows the number of kilocalories burned in one minute by a person weighing 50 kg for several different activities.

Activity	Number of kilocalories burned *per minute* by a 50-kg person
Basketball	6.9
Card playing	1.25
Dancing	3.75
Football	6.6
Jumping rope	8.1
Playing the piano	2
Painting	1.7
Walking	4

2. Use proportions to calculate the number of minutes it would take to burn off 50 kilocalories while playing basketball. What shortcut could be used to calculate the number of minutes it takes to burn off a given number of kilocalories?

$$\frac{\text{kilocalories}}{\text{minutes}} \qquad \frac{6.9}{1} = \frac{50}{x} \qquad 6.9x = 50 \qquad x = 7.25 \text{ min}$$

Shortcut would be to divide the number of kilocalories by the kilocalories burned per minute from the table.

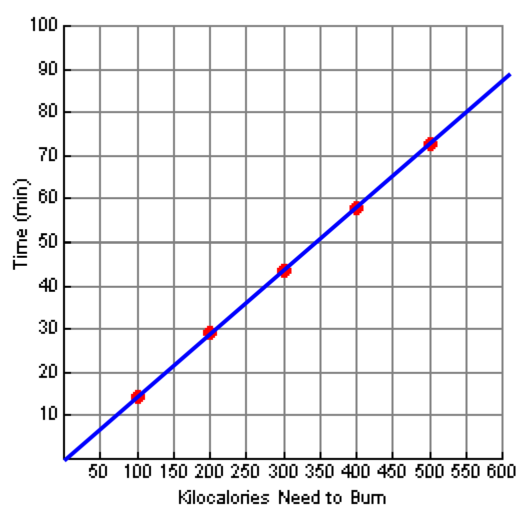
Burning Calories (p. 2 of 3) KEY

3. Choose one activity. Record the name of your activity. Calculate the number of minutes required to burn 100 kcal, 200 kcal, 300 kcal, 400 kcal, and 500 kcal rounded to the nearest tenth of a minute.

Sample table

Energy Burned (kcal)	Activity **Basketball** (min)
100	14.5
200	29
300	43.5
400	58
500	72.5

4. Create a scatterplot for the activity below. Graph the number of kilocalories along the *x*-axis and time in minutes along the *y*-axis. Draw a smooth line through the points on the scatterplot. Sample graph based on table above.



Burning Calories (p. 3 of 3) KEY

5. Describe any patterns in the table and scatterplot.
   Sample answers: The points are in a straight line, the scatterplot is increasing, as the number of kilocalories increases, the number of minutes increases.

6. Use the table and graph to predict the number of minutes it would take to burn 800 kcal for the activity you chose. How do your results compare with others in the class that selected the same activity?
   Students can extend the lines on the graph or the values in the table to make the predictions. Students should get answers that are close to others that select the same activities.

7. How does the number of minutes change as the number of kilocalories increases? How do the table and scatterplot support your answer?
   The number of minutes increases as the number of kilocalories increases. The values in the table increase and the scatterplot goes up from left to right.

8. How are the scatterplots created in **Miles and Intersections** and **Burning Calories** the same? How are they different?
   They are the same in that they show a relationship between two quantities, have scales, etc. They are different in that Burning Calories shows a pattern, is linear, and can be used to make predictions and the other cannot.

9. If you were able to make a prediction of the number of minutes it would take to burn 800 kcal, how is this situation different from that in **Miles and Intersections** where you were asked to predict the number of intersections if you lived 7 miles from school?
   In the kilocalorie problem, there was a pattern. The number of minutes could actually be calculated. The lines went through all the points and could be extended.

Keep your work from this exploration!

Burning Calories (p. 1 of 3)

In **Miles and Intersections**, you learned that two quantities such as (*distance, number of intersections*) is a **relation** as long as it forms a set of ordered pairs**.** You also learned that when there is no pattern in the data, it is difficult to make predictions. What if there is a pattern? What if changing one quantity changes the other quantity in a predictable way? In this exploration, you will investigate another relation with special characteristics.

The food you eat provides your body the energy it needs to maintain body functions such as temperature regulation, blood circulation, bone growth and muscle repair. It also provides the energy needed to be physically active. When you engage in physical activity, you burn ***kilocalories***. A kilocalorie (kcal) is the amount of energy required to raise the temperature of one liter (1 L) of water one degree Celsius (1° C). A dietary Calorie (with a capital *C*) is equal to one kilocalorie.

1. Imagine you are going to the gym after school. You have had extra snacks during the day. Does the number of minutes you need to exercise depend on the number of kilocalories you want to work off? Explain why or why not.

   The table below shows the number of kilocalories burned in one minute by a person weighing 50 kg for several different activities.

Activity	**Number of kilocalories burned *per minute* by a 50-kg person**
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Walking	4

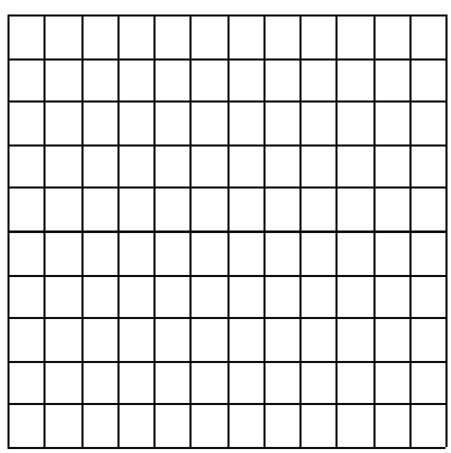
2. Use proportions to calculate the number of minutes it would take to burn off 50 kilocalories while playing basketball. What shortcut could be used to calculate the number of minutes it takes to burn off a given number of kilocalories?

Burning Calories (p. 2 of 3)

3. Choose one activity. Record the name of your activity. Calculate the number of minutes required to burn 100 kcal, 200 kcal, 300 kcal, 400 kcal, and 500 kcal rounded to the nearest tenth of a minute.

Energy Burned (kcal)	**Activity** ______________ **(min)**
100	
200	
300	
400	
500	

4. Create a scatterplot for the activity below. Graph the number of kilocalories along the *x*-axis and time in minutes along the *y*-axis. Draw a smooth line through the points on the scatterplot.



Burning Calories (p. 3 of 3)

5. Describe any patterns in the table and scatterplot.

6. Use the table and graph to predict the number of minutes it would take to burn 800 kcal for the activity you chose. How do your results compare with others in the class that selected the same activity?

7. How does the number of minutes change as the number of kilocalories increases? How do the table and scatterplot support your answer?

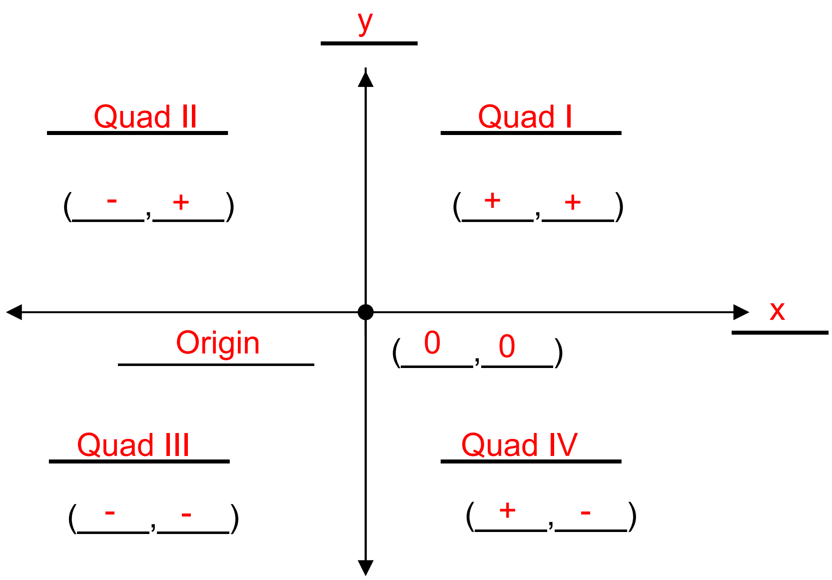
8. How are the scatterplots created in **Miles and Intersections** and **Burning Calories** the same? How are they different?

9. If you were able to make a prediction of the number of minutes it would take to burn 800 kcal, how is this situation different from that in **Miles and Intersections** where you were asked to predict the number of intersections if you lived 7 miles from school?

Keep your work from this exploration!

Cartesian Coordinate System (pp. 1 of 3) KEY

Label the parts of the Cartesian Coordinates System below with the following: x-axis, y-axis, and Quadrant I, Quadrant II, Quadrant III, Quadrant IV, origin, the coordinates of the origin, and when x is positive or negative, and when y is positive or negative in the ordered pair (+, +).



The Cartesian coordinate system is used to graph relationships between quantities. It is composed of two number lines called the x-axis and the y-axis. These two number lines divide the plane into four quadrants.

- A point or ordered pair is written as (x, y) or (x, f(x)) and can be located in any quadrant or on the x-axis or y-axis. NOTE: Another way to write y is f(x).

  $\{(-2,3),(0,0),(2,-5)\}$ Set of ordered pairs

- Relations can be graphed as a point or a set of points.
  For the set of ordered pairs, in which quadrant would each point be located?
  (-2 , 3) Quadrant II
  (0, 0) Origin, intersection of the x and y axes
  (2, -5) Quadrant IV
  (0, 3) Not in a quadrant but on the y-axis

- The domain of the relationship is the set of permissible x values. The notation for domain is D:{-2, 0, 2}. Domains can be continuous or discrete.
  - Discrete data is individual points that would not be connected when graphed because not all rational values define the domain. (Connected with a broken line on a graph, D: {-2, 0, 2})
  - Continuous data is an infinite number of points that are connected when graphed because all real values can be defined in the domain. (Connected with a solid line on a graph, D: {-2 < x < 2} or {x | -2 < x < 2})

- The range of the relationship is the set of permissible y values. The notation for range is R: {3, 0, -5} or R: {-2 < y < 2} or {y | -2 < y < 2}.

Cartesian Coordinate System (pp. 2 of 3) KEY

- Relations in which each element of the domain is paired with exactly one element of the range are called functions.
  - If a set of data is a function, each *x* value is paired with a unique *y* value, i.e., the x's do not repeat.
  - If a set of data is a function, a vertical line will cross the graph of the data in only one point.

Function Analogy: Consider the domain to be a set of people on a bus. Think of each bus stop along the way as the range. The "function" of the bus is to deliver people to their respective destinations. It is possible for two or more people to get off at one bus stop (y), however, it is not possible for the same person (x) to get off at two different bus stops. A person (x), is associated with only one bus stop (y).

- If the *y* value increases as the *x* value increases, the function is increasing. On the graph an increasing function will go up from left to right.
- If the *y* value decreases as the *x* value increases, the function is decreasing. On the graph a decreasing function will go down from left to right.

Connections

1. Remember that a relation between two quantities is a set of ordered pairs of the form (*x, y*).
   a. In **Miles and Intersections** is there a **relation** between miles to school and intersections crossed? Explain why or why not.
   Yes, the data in the table can be written as ordered pairs (*miles, intersections*) and graphed on a coordinate plane.

   b. In **Burning Calories** is there a **relation** between kilocalories burned and minutes? Explain why or why not.
   Yes, the data in the table can be written as ordered pairs (*number of kcal, number of minutes*) and graphed on a coordinate plane.

   c. *Relation* and *dependence* both describe how two quantities can be connected. How are the two ideas different?
   Two quantities can be related without one being dependent on the other.

Cartesian Coordinate System (pp. 3 of 3) KEY

2. The number of minutes ***depends*** on the number of kcal. When calculating the number of minutes, changing the number of kcal changed the answer. Did the number of intersections depend on the distance? Why or why not?
   No. The distance didn't affect the number of intersections.

3. Identify the independent and dependent variables in the activity **Burning Calories**.
   Independent – kilocalories Dependent – minutes of exercise

4. Give another example of a situation where one quantity depends on another. (*Example: The amount I earn depends on the number of hours I work.*)
   Answers vary.

5. Which of the two previous activities represents a functional relationship? Explain your reasoning.
   Burning Calories is a functional relationship. Each x value (number of kilocalories) is assigned a unique y value (minutes of exercise). It also passes the vertical line test. The Miles and Intersections is not a function because the x values (miles) may be associated with more than one y value (intersection).

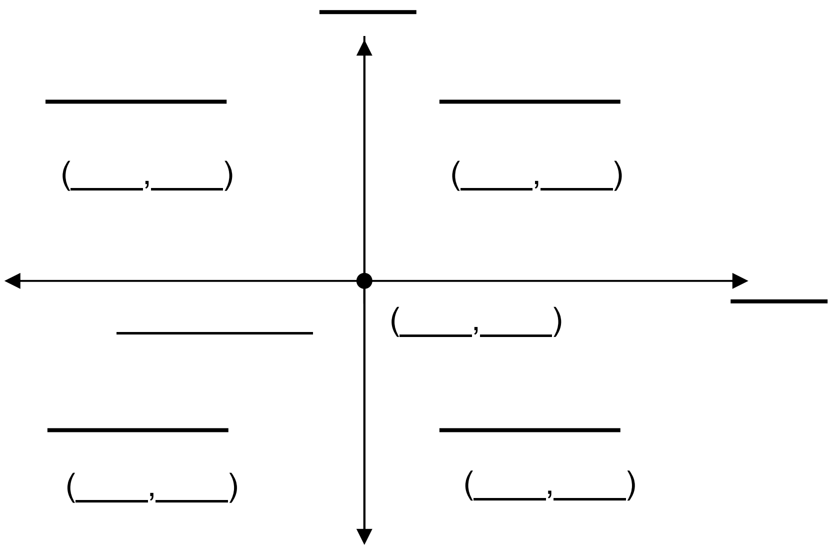
6. Does **Burning Calories** represent a continuous or discrete domain? Explain your reasoning.
   The domain is continuous, because it could be any real value including fractions and decimals.

7. What is the domain and range of the relation investigated in **Burning Calories**?
   D: $\{x \mid x \geq 0\}$ R: $\{y \mid y \geq 0\}$

8. Is the relation in **Burning Calories** increasing or decreasing? Explain your reasoning.
   Burning Calories is an increasing functional relationship. As the x value (number of kilocalories) increases the y value (minutes of exercise) increases. The graph of the function goes up from left to right.

Cartesian Coordinate System (pp. 1 of 3)

Label the parts of the Cartesian Coordinates System below with the following: x-axis, y-axis, and Quadrant I, Quadrant II, Quadrant III, Quadrant IV, origin, the coordinates of the origin, and when x is positive or negative, and when y is positive or negative in the ordered pair (+, +).



The ____________________________ is used to graph relationships between quantities. It is composed of two number lines called the x-axis and the y-axis. These two number lines divide the plane into four quadrants.

- A __________ or _______________ is written as (x, y) or (x, f(x)) and can be located in any quadrant or on the x-axis or y-axis. NOTE: Another way to write y is f(x).

  $\{(-2,3),(0,0),(2,-5)\}$ Set of ordered pairs

- __________ can be graphed as a point or a set of points.
  For the set of ordered pairs, in which quadrant would each point be located?
  (-2 , 3) __________
  (0, 0) _____________________________
  (2, -5) __________
  (0, 3) _____________________________

- The __________ of the relationship is the set of permissible x values. The notation for domain is D:{-2, 0, 2}. Domains can be continuous or discrete.
  - __________ data is individual points that would not be connected when graphed because not all rational values define the domain. (Connected with a broken line on a graph, D: {-2, 0, 2})
  - __________ data is infinite number of points that are connected when graphed because all real values can be defined in the domain. (Connected with a solid line on a graph, D: {-2 < x < 2} or {x | -2 < x < 2})

- The __________ of the relationship is the set of permissible y values. The notation for range is R: {3, 0, -5} or R: {-2 < y < 2} or {y | -2 < y < 2}.

Cartesian Coordinate System (pp. 2 of 3)

- Relations in which each element of the domain is paired with exactly one element of the range are called ________________.
    - o If a set of data is a function, ____
      ____
    - o If a set of data is a function, ____
      ____

Function Analogy: Consider the domain to be a set of people on a bus. Think of each bus stop along the way as the range. The "function" of the bus is to deliver people to their respective destinations. It is possible for two or more people to get off at one bus stop (y), however, it is not possible for the same person (x) to get off at two different bus stops. A person (x), is associated with only one bus stop (y).

- If the *y* value increases as the *x* value increases, the function is ________________. On the graph an increasing function will go up from left to right.
- If the *y* value decreases as the *x* value increases, the function is ________________. On the graph a decreasing function will go down from left to right.

Connections

1. Remember that a relation between two quantities is a set of ordered pairs of the form (*x, y*).
    a. In **Miles and Intersections** is there a **relation** between miles to school and intersections crossed? Explain why or why not.

    b. In **Burning Calories** is there a **relation** between kilocalories burned and minutes? Explain why or why not.

    c. *Relation* and *dependence* both describe how two quantities can be connected. How are the two ideas different?

Cartesian Coordinate System (pp. 3 of 3)

2. The number of minutes ***depends*** on the number of kcal. When calculating the number of minutes, changing the number of kcal changed the answer. Did the number of intersections depend on the distance? Why or why not?

3. Identify the independent and dependent variables in the activity **Burning Calories**.

4. Give another example of a situation where one quantity depends on another. (*Example: The amount I earn depends on the number of hours I work.*)

5. Which of the two previous activities represents a functional relationship? Explain your reasoning.

6. Does **Burning Calories** represent a continuous or discrete domain? Explain your reasoning.

7. What is the domain and range of the relation investigated in **Burning Calories**?

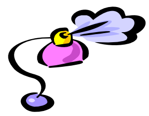
8. Is the relation in Burning Calories increasing or decreasing? Explain your reasoning.

Independent/Dependent Sentence Strips Connections KEY

1. The quality of a music performance is related to the amount of practice.

Practice time  Performance 

2. The amount of perfume/cologne applied is related to the severity of an allergic reaction.

Amount of perfume/cologne  Allergic reaction 

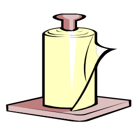
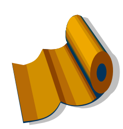
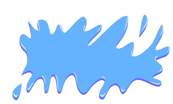
3. The effects of hyperactivity are related to the amount of caffeine consumed.

Caffeine consumed  Hyperactivity 

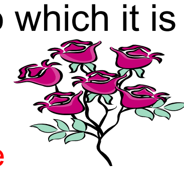
4. The amount of study time is related to a test grade.

Amount of study time  Test grade 

5. The amount of liquid picked up relates to the absorbency of different paper towel brands.

Absorbency  Liquid picked up  

6. The rate of plant growth is related to the color of light to which it is exposed.

Color of light  Plant growth rate 

7. Write one more cause and effect relationship. Identify the independent and dependent quantities or attributes.

   Will vary

Independent/Dependent Sentence Strips Connections (pp. 1 of 2)

Answer Chart

Independent Variable	Dependent Variable

Independent/Dependent Sentence Strips Connections (pp. 2 of 2)

Rewrite each of the following as a statement of one attribute depending on the other—a verbal statement of the relationships you created with the sentence strips.

1. The quality of a music performance is related to the amount of practice.

2. The amount of perfume/cologne applied is related to the severity of an allergic reaction.

3. The effects of hyperactivity are related to the amount of caffeine consumed.

4. The amount of study time is related to a test grade.

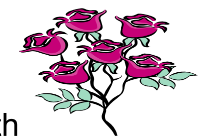
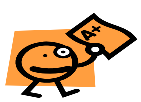
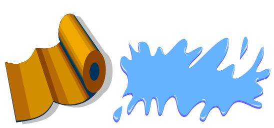
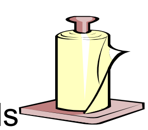
5. The amount of liquid picked up relates to the absorbency of different paper towel brands.

6. The rate of plant growth is related to the color of light to which it is exposed.

7. Write one more cause and effect relationships. Identify the independent and dependent variables.

Independent/Dependent Sentence Strips Cards

Cut out the word strips. Use a glue stick to attach independent variables on the left and the related dependent variable on the right in the answer chart on the Connections pages.

Quality of Performance 	Allergic reaction 
Plant growth 	Perfume/cologne 
Hyperactivity 	Study time 
Test grade 	Practice time 
Liquid picked up 	Caffeine consumed 
Absorbency of paper towels 	Color of light 

Relations and Dependency (pp. 1 of 4) KEY

A mathematical relation expresses a dependent relationship where one quantity depends in a systematic way on another quantity.

In some cases there is a cause and effect relationship where the cause is the independent variable and the effect is the dependent variable.

1. Example: Lake Travis will rise 2 feet if it rains 15 inches in the watershed.

In other cases there is not a cause and effect relationship, but there can still be an independent/dependent relationship. In this type of relationship either can be the independent variable, which then forces the other to be dependent.

2. Example: Henry has an arm span of 64 inches and a height of 66 inches.

Some are generalized algebraic relationships.

3. Example: $y = 2x + 1$ is a function and expresses a dependency relationship.

Input Independent Domain x	**Output Dependent Range** y
-5	-9
-3	-5
-1	-1
0	1
1	3
3	7
5	11

Relations and Dependency (pp. 2 of 4) KEY

- The value of *y* depends on the value of *x*.
- The variable *x* is called the input or independent variable. The set of permissible values for the independent variable is called the domain.
- The variable y is called the output or dependent variable. The set of permissible values for the dependent variable is called the range.

Example	Independent	Dependent	Domain (value)	Range (value)
1. Lake Travis will rise 2 feet if it rains 15 inches in the watershed.	Amount of rain	Raising water	15	2
2. Henry has an arm span of 64 inches and a height of 66 inches.	Arm span OR height	Height OR Arm Span	64 OR 66	66 OR 64
3. $y = 2x + 1$ is a function and expresses a dependency relationship.	x	y	"R" all real numbers	"R" all real number

Relations and Dependency (pp. 3 of 4) KEY

Practice Problems

1. Sue Ann received a statement from her bank listing the balance in her money market account for the past four years. What is the independent quantity in this table?

Time	Balance
0	$1000
1	$1240
2	$1390
3	$1450
4	$1622

Time (or years) is independent; balance is dependent.

2. Garrett is in charge of making 120 corsages for homecoming. He decides to ask some of his classmates for help. The number of corsages each person can make can be represented by the function $f(h)=\frac{120}{h+1}$ where h is the number of classmates that help Garrett make corsages. Which is the dependent quantity of this function?
   Dependent is corsages; independent is friends.

3. The TM Tennis Team played a total of 162 matches last season. The number of matches the team lost, l, and the number of matches the team won, w, are represented by the formula below. What quantity does the dependent variable represent?
   $$l = 162 - w$$
   Losses is dependent; wins is independent.

4. Pat hikes at an average rate of four miles per hour. The number of miles, m, she hikes is viewed as a function of the number of hours, h, she hikes. What is the independent variable?
   Hours is independent; miles is dependent.

5. A taxi driver charges an initial fee of $5.00 plus $0.50 per mile. What is the independent variable quantity in this situation?
   Number of miles driven is independent; total cost is dependent.

6. A long distance telephone company charges $2.95 per month and $0.18 per minute for phone calls. What is the dependent variable quantity in this situation?
   Total bill is dependent; number of minutes is independent.

7. A plumber charges forty dollars to make a house call plus thirty-five an hour for labor. What are the independent and dependent variables?
   Hours worked is independent; total bill is dependent.

Relations and Dependency (pp. 4 of 4) KEY

8. The table below represents the relationship between the number of gallons of gas in a gas tank and the number of miles that can be driven. Which quantity represents the dependent quantity in this table?

Gas in Tank (gallons)	Miles that Can Be Driven
0	0
1	23
2	46
3	69
4	115
5	184

Total miles driven is dependent; amount of gas in gallons is independent.

9. Larrissa answered all twenty-five questions on a multiple-choice history exam. Her score was computed by multiplying the number of wrong answers by four and then subtracting the number from one hundred. What quantity represents the independent variable?
Number of problems wrong is independent; grade is dependent.

10. The cost for copying a document is a function of the number of pages in the document. In this situation, what is the dependent variable?
Total cost is dependent; number of pages copied is independent.

11. Charles partially filled a container with sand. The container was shaped like a box and had dimensions 2 feet long, 1.5 feet wide, and 6 inches high. If w represents the height of the sand (in inches), and the volume V (in cubic inches) of the sand is given by the formula $\boldsymbol{V = 3w}$, which quantity is the independent variable?

   a. The height of the container
   b. The volume of the container
   c. The height of the sand in the container
   d. The volume of the sand in the container

12. In the situation below, there are three functional relationships. Identify **at least one** independent and dependent relationship. In that relationship, tell which one is the independent variable and which one is the dependent variable.

    The monthly cost of electricity for a home is based on the number of kilowatt-hours (kwh) of electricity used. The number of kilowatt hours used is based on the number of watts of electricity each light bulb or appliance uses and the amount of time it is used.

    Kilowatts independent, cost dependent
    Watts independent, kilowatts dependent
    Time independent, kilowatts dependent

Relations and Dependency **(pp. 1 of 4)**

A mathematical relation expresses a dependent relationship where one quantity depends in a systematic way on another quantity.

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2. Example: Henry has an arm span of 64 inches and a height of 66 inches.

Some are generalized algebraic relationships.

3. Example: $y = 2x + 1$ is a function and expresses a dependency relationship.

Input **Independent** **Domain** x	**Output** **Dependent** **Range** y

Relations and Dependency (pp. 2 of 4)

- The value of ____.
- The variable x is called the _______________or ___________________________. The set of permissible values for the independent variable is called the ________________.
- The variable y is called the _______________or ___________________________. The set of permissible values for the dependent variable is called the ________________.

Example	Independent	Dependent	Domain (value)	Range (value)
1. Lake Travis will rise 2 feet if it rains 15 inches in the watershed.				
2. Henry has an arm span of 64 inches and a height of 66 inches.				
3. $y = 2x + 1$ is a function and expresses a dependency relationship.				

Relations and Dependency (pp. 3 of 4)

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Relations and Dependency (pp. 4 of 4)

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    C. The height of the sand in the container
    D. The volume of the sand in the container

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    The monthly cost of electricity for a home is based on the number of kilowatt-hours (kwh) of electricity used. The number of kilowatt hours used is based on the number of watts of electricity each light bulb or appliance uses and the amount of time it is used.

Facts About Functions (pp. 1 of 7) KEY

1. Identify which relationships are functional and explain your reasoning.

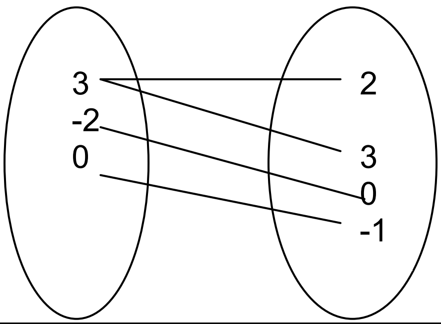
a. {(3,2), (3,3), (-2,0), (0,-1)}

Non-function, the x-coordinate of 3 is paired with both a 2 and a 4.

b. {(3,2), (4,2), (-2,3), (0,0)}

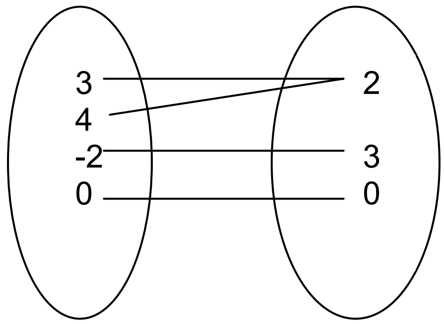
Function, each x-coordinate occurs only once and is paired with only one y-coordinate.

c.



Non-function, the 3 in the domain is paired with two range values.

d.



Function, each element of the domain is paired with only one element of the range.

e.

-2	0
0	-1
3	2
3	3

Non-function, the 3 in the domain is paired with two range values.

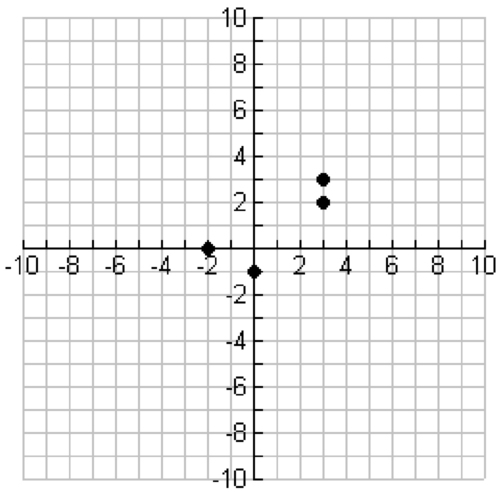
f.

-2	3
0	0
3	2
4	2

Function, each element of the domain is paired with only one element of the range.

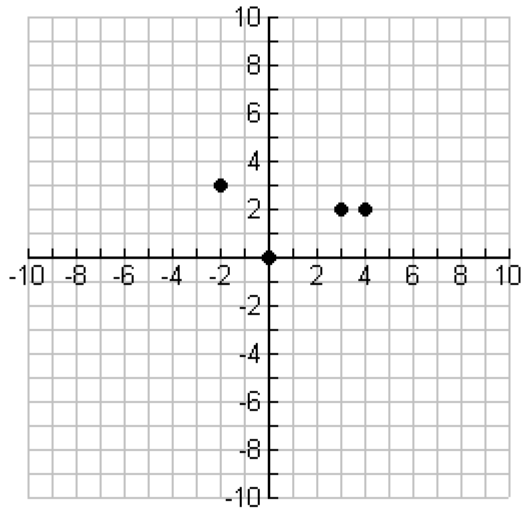
Facts About Functions (pp. 2 of 7) KEY

g.



Non-function, the 3 in the domain is paired with two range values.

h.



Function, each element of the domain is paired with only one element of the range.

2. For a set of points, determine if it is a function and identify the domain and range.
   {(-4, 5), (5, 6), (0, -7), (1, 0), (8, 9), (-2, -2)}
   Function
   Domain: {-4,-2, 0, 1, 5, 8}
   Range: {-7,-2, 0, 5, 6, 9}

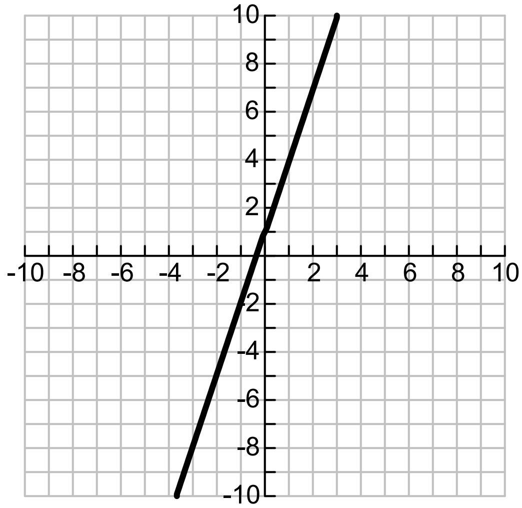
3. For a table of data, determine if it is a function, identify the independent and dependent variable, and state the domain and range.

Seconds (x)	**Temperature (y)**
2	0
4	-2
6	-4
8	-6
10	-8

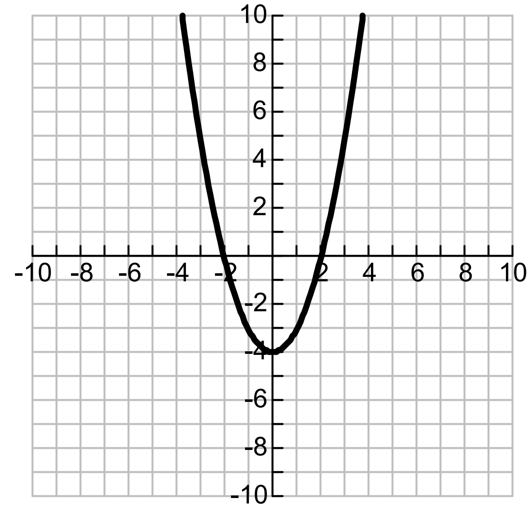
Function
Independent variable: Seconds
Dependent variable: Temperature
Domain: {2, 4, 6, 8, 10}
Range: {0,-2,-4,-6,-8}

Facts About Functions (pp. 3 of 7) KEY

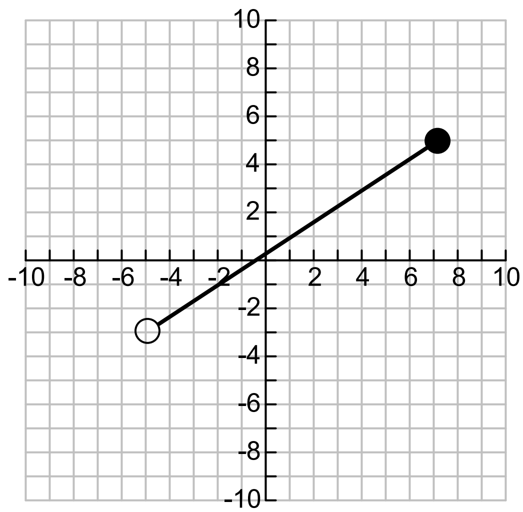
4. For given graphs, determine if it is a function and identify the domain and range.



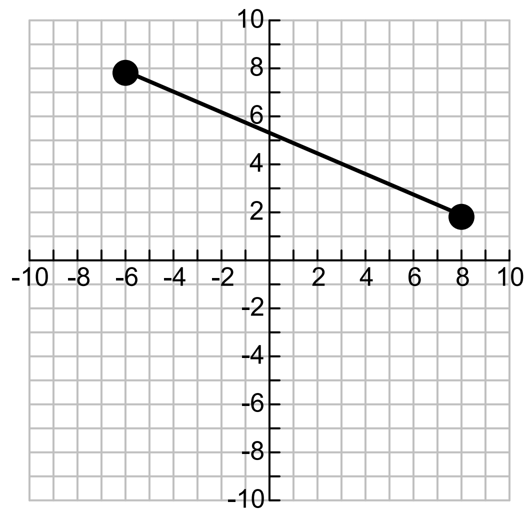
Domain: all real numbers
Range : all real numbers
Function



Domain: all real numbers
Range: $\{y \geq -4\}$
Function



Domain: $\{-5 < x \leq 7\}$
Range: $\{-3 < y \leq 5\}$
Function

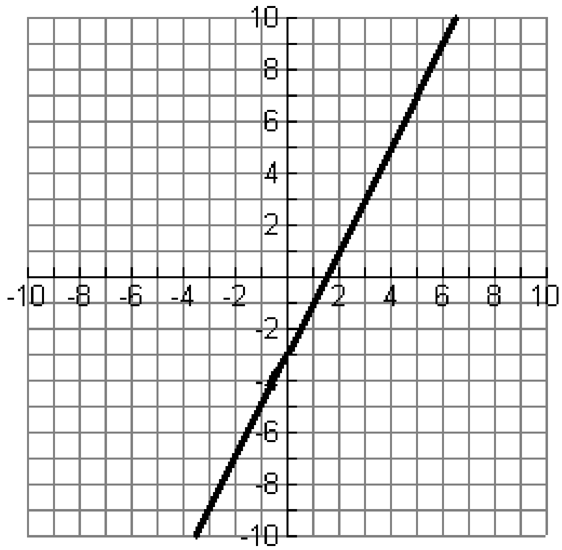


Domain: $\{-6 \leq x \leq 8\}$
Range: $\{2 \leq y \leq 8\}$
Function

Facts About Functions (pp. 4 of 7) KEY

5. For a relation (y =), determine if it is a function, identify the independent and dependent variable, and state the domain and range. Graph the relation.

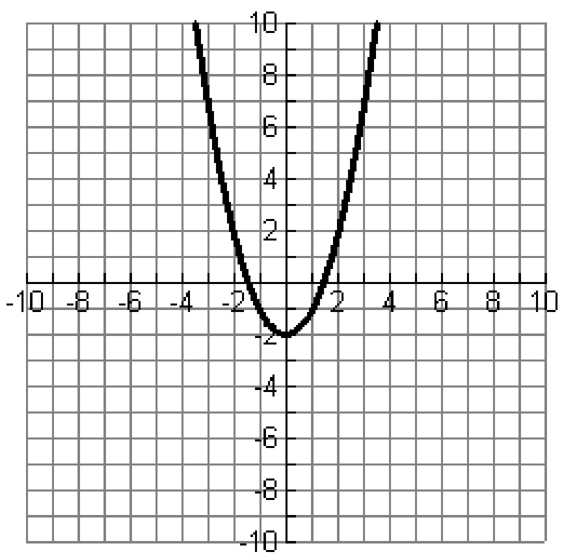
a. $y = 2x - 3$



-3	-9
-2	-7
-1	-5
0	-3
1	-1
2	1

Independent is x
D: {R}
Dependent is y
R: {R}
Function - linear

b. $y = x^2 - 2$



-3	7
-2	2
-1	-1
0	-2
1	-1
2	2
3	7

Independent is X
D: {R}
Dependent is Y
R :{ $y \geq -2$}
Function
Non-linear

Facts About Functions (pp. 5 of 7) KEY

Practice Problems

1. Compare and contrast the characteristics of relations and functions. Study the statements below that are about relations and functions. Place a check mark in the appropriate boxes if the statement is true for all relations, or if the statement is true for all functions.

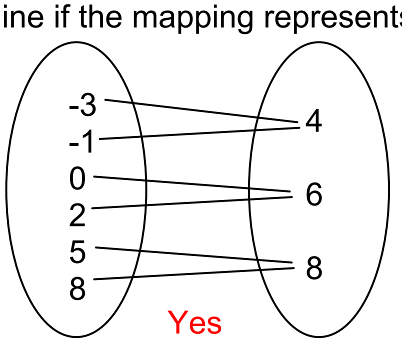
Statement	Attribute of All Relations	Attribute of All Functions
It can be discrete or continuous.	✓	✓
It has a domain and range.	✓	✓
It can be represented by ordered pairs in the form (*x, y*).	✓	✓
It matches exactly one independent value with each dependent value.		✓
It can be represented by a graph.	✓	✓
It can be represented by a table.	✓	✓

a. What conjecture can you make about relations and functions? Fill in each blank with the word *relation* or *function* to make a true statement.

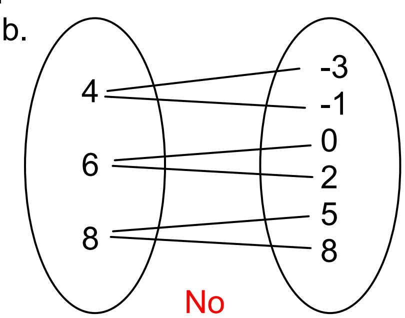
A function is always a relation, but a relation is not always a function.

b. What is the defining characteristic of functions? In other words, what makes a relation a function?
A relation is a function when only one y-value is matched with each x-value.

2. Determine if the mapping represents a function.

a.


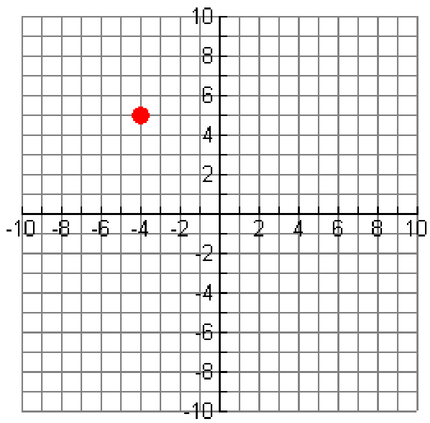
Yes

b.


No

Facts About Functions (pp. 6 of 7) KEY

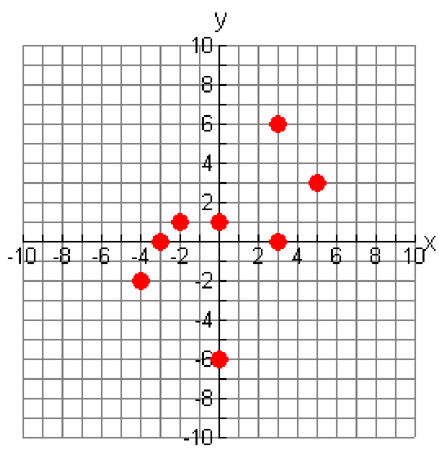
3. Plot the point (-4, 5). Identify the domain and range.



D:{-4}, R:{5}

4. Identify the domain and range. Determine if it is a function and tell why.

a. Plot the set of points {(3, 0), (-2, 1), (0, -6), (-3, 0), (-4, -2), (0, 1), (5, 3), (3, 6)}.

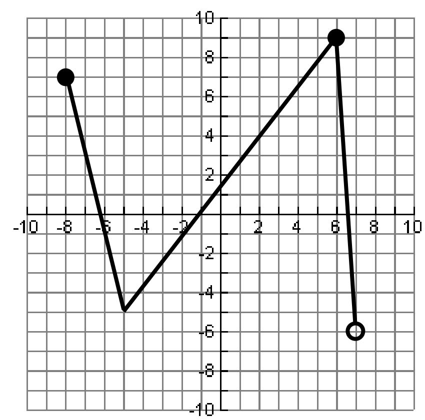


D :{ -4, -3,-2, 0, 3, 5}

R:{-6,-2, 0, 1, 3}

Not a function domain value 0 repeats

b.



D: $\{x \mid -8 \le x < 7\}$

R: $\{y \mid -6 < y \le 9\}$

Is a function, passes the vertical line test.

Facts About Functions (pp. 7 of 7) KEY

5. Make a table of values and plot the relationship y = 2x + 1. Identify the domain and range. Determine if it is a function and tell why.

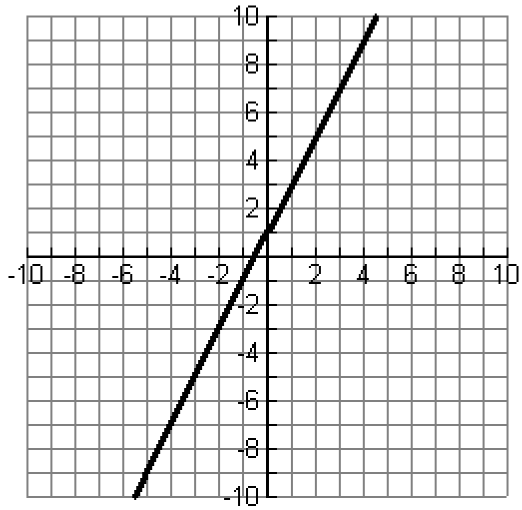


Table will vary.
D: {All real numbers} or D: {R}
R: {All real numbers} or R: {R}
Function, passes vertical line test, domains do not repeat.

6. Make a table of values and plot the relationship $y = x^2 + 1$. Identify the domain and range. Determine if it is a function and tell why.

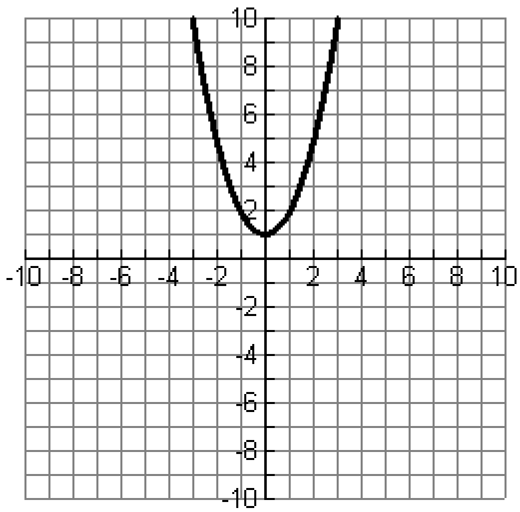


Table will vary.
D: {R}
R: {R ≥ 1}
Function because x doesn't repeat

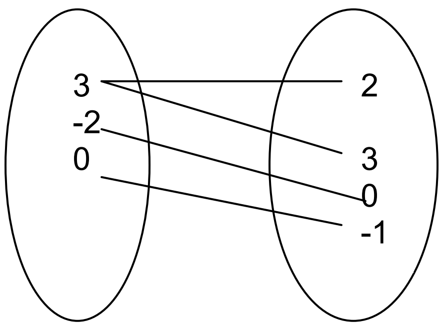
Facts About Functions (pp. 1 of 7)

1. Identify which relationships are functional and explain your reasoning.

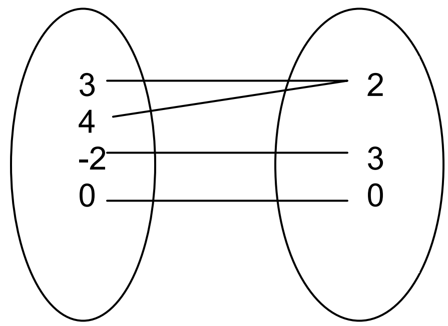
a. {(3,2),(3,3),(-2,0),(0,-1)}

b. {(3,2),(4,2),(-2,3),(0,0)}

c.



d.



e.

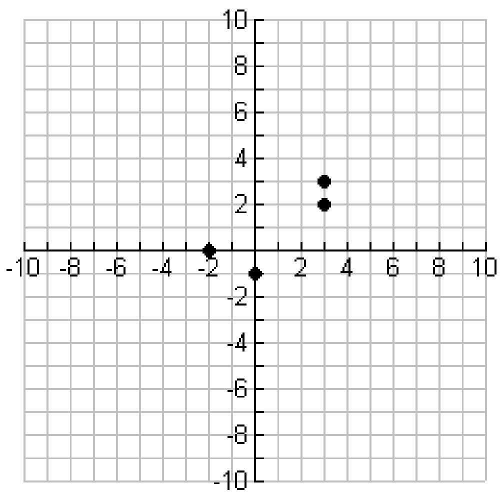
-2	0
0	-1
3	2
3	3

f.

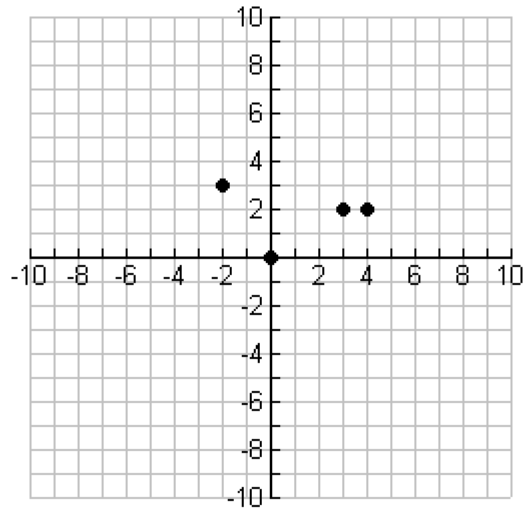
-2	3
0	0
3	2
4	2

Facts About Functions (pp. 2 of 7)

g.



h.



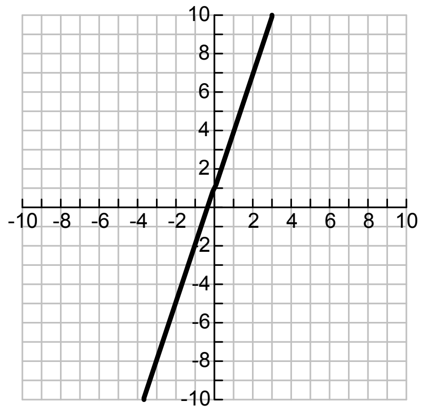
2. For a set of points, determine if it is a function and identify the domain and range.
   {(-4, 5), (5, 6), (0, -7), (1, 0), (8, 9), (-2, -2)}

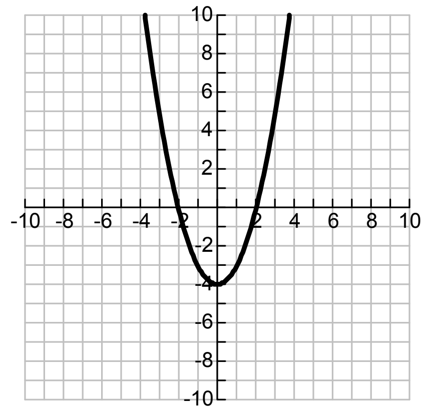
3. For a table of data, determine if it is a function, identify the independent and dependent variable, and state the domain and range.

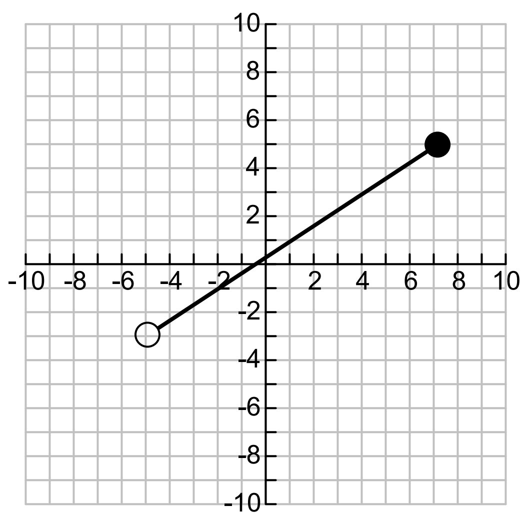
Seconds (x)	**Temperature (y)**
2	0
4	-2
6	-4
8	-6
10	-8

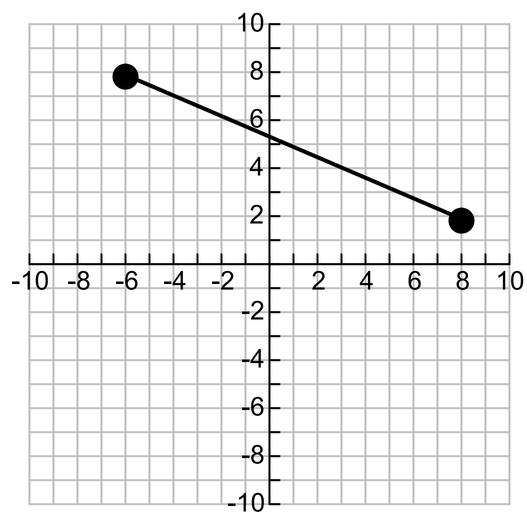
Facts About Functions (pp. 3 of 7)

4. For given graphs, determine if it is a function and identify the domain and range.





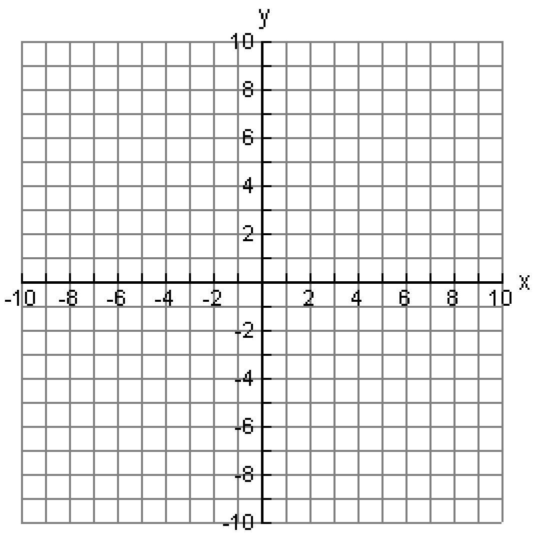




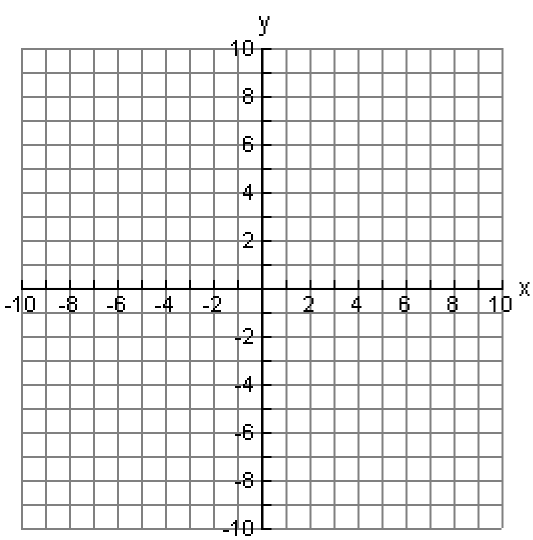
Facts About Functions (pp. 4 of 7)

5. For a relation (y =), determine if it is a function, identify the independent and dependent variable, and state the domain and range. Graph the relation.

   a. $y = 2x - 3$



   b. $y = x^2 - 2$



Facts About Functions (pp. 5 of 7)

Practice Problems

1. Compare and contrast the characteristics of relations and functions. Study the statements below that are about relations and functions. Place a check mark in the appropriate boxes if the statement is true for all relations, or if the statement is true for all functions.

Statement	Attribute of All Relations	Attribute of All Functions
It can be discrete or continuous.		
It has a domain and range.		
It can be represented by ordered pairs in the form (*x, y*).		
It matches exactly one independent value with each dependent value.		
It can be represented by a graph.		
It can be represented by a table.		

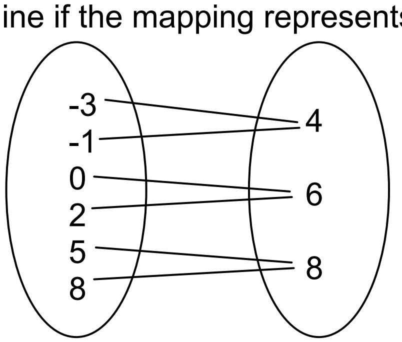
a. What conjecture can you make about relations and functions? Fill in each blank with the word *relation* or *function* to make a true statement.

A __________ is always a __________, but a __________ is not always a __________.

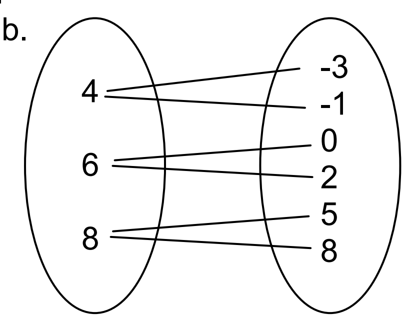
b. What is the defining characteristic of functions? In other words, what makes a relation a function?

2. Determine if the mapping represents a function.

a.

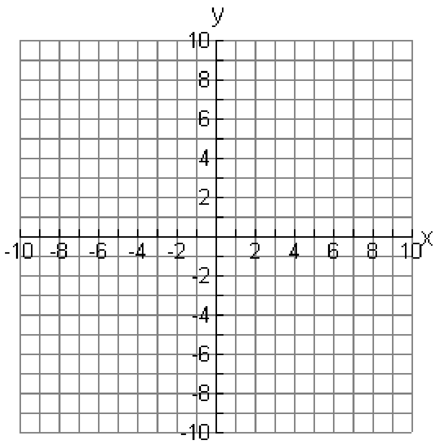


b.



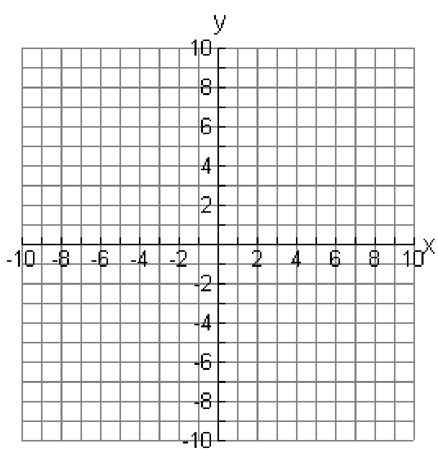
Facts About Functions (pp. 6 of 7)

3. Plot the point (-4, 5). Identify the domain and range.

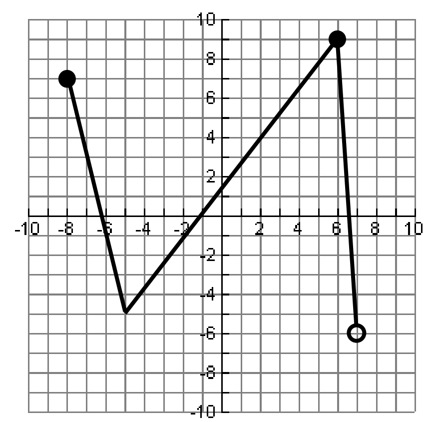


4. Identify the domain and range. Determine if it is a function and tell why.

a. Plot the set of points
{(3, 0), (-2, 1), (0, -6), (-3, 0), (-4, -2), (0, 1), (5, 3), (3, 6)}.

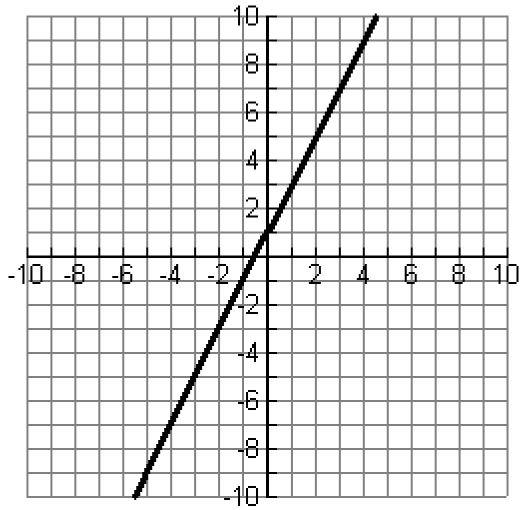


b.

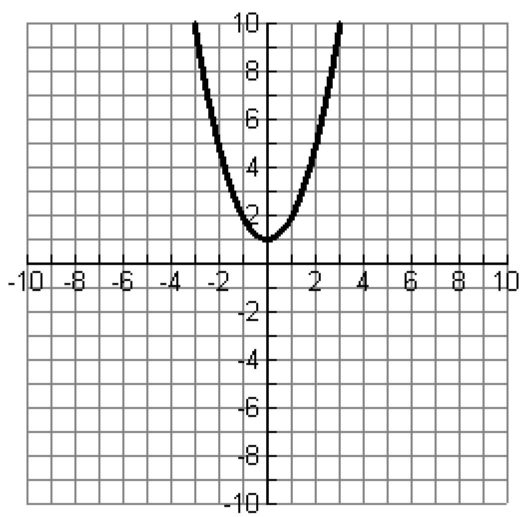


Facts About Functions (pp. 7 of 7)

5. Make a table of values and plot the relationship y = 2x + 1. Identify the domain and range. Determine if it is a function and tell why.



6. Make a table of values and plot the relationship y = x^2 + 1. Identify the domain and range. Determine if it is a function and tell why.



Ticket Prices (pp. 1 of 4) KEY

Does the *number* of concert tickets sold depend on the *price* of the ticket? If one variable depends on another, can there be more than one dependent variable for each independent variable? In this exploration, you will further investigate the idea of dependence between two quantities and functional relationships.

A popular band is scheduled to play at the Starplex Amphitheater; however, the amphitheater's management and the band cannot come to an agreement about the price of the tickets.

1. The amphitheater, which seats 50,000, has seen a drop of 150 tickets sold for each dollar increase in ticket price. The number of tickets sold can be calculated using the formula

$$t = 50000 - 150p,$$

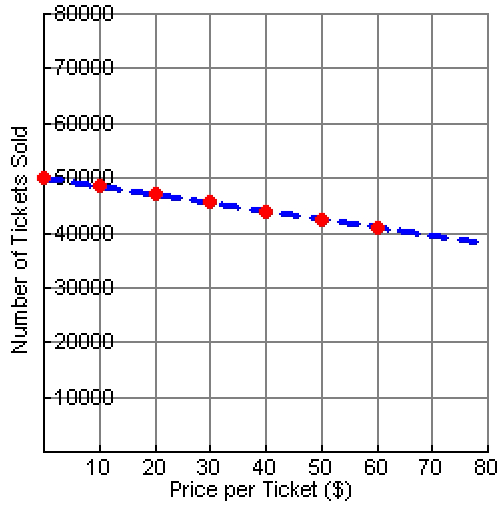
where t represents the number of tickets sold and p represents the price per ticket in dollars.

Complete the table.

Price per ticket in dollars (p)	**Process**	**Number of tickets sold (t)**
0	50000 – 150(0)	50,000
10	50000 – 150(10)	48,500
20	50000 – 150(20)	47,000
30	50000 – 150(30)	45,500
40	50000 – 150(40)	44,000
50	50000 – 150(50)	42,500
60	50000 – 150(60)	41,000
p	50000 – 150(p)	50000 – 150p

Ticket Prices (pp. 2 of 4) KEY

2. Create a scatterplot. Graph the price in dollars along the *x*-axis and the number of tickets sold along the *y*-axis.



3. How did you determine the number of tickets sold?
   Sample answers: I multiplied the number of tickets by 150 and subtracted that answer from 50000. Some students might recognize and use the pattern in the table. Some might repeatedly subtract 150 from the number of tickets to find the next answer.

4. When you calculated the number of tickets sold, how many different answers did you get for each different ticket price?
   Just one.

5. How is your answer to question 4 reflected in the graph?
   On the graph each ticket price corresponds to one and only one point on the line representing the number of tickets sold.

6. What are the independent and dependent variables in the problem situation?
   Independent – price per ticket   Dependent – Number of tickets sold

7. Does this situation represent discrete or continuous data? Why?
   Discrete data because you cannot have part of a ticket

8. What happens to the number of tickets sold as the price increases? How is this reflected in the table and graphs?
   The number of tickets sold decreases. In the table, the number of tickets sold gets smaller, and in the graphs, the points and the line go down from left to right.

Ticket Prices (pp. 3 of 4) KEY

9. Predict the price at which no tickets will be sold.
   Around $333

10. How did you determine the solution to question 9?
    Answers vary. Students might repeatedly subtract 150 until the answer is zero, divide 51,500 by 150 or extend the table or graph

11. Does the number of tickets sold depend on the price? Why or why not?
    Yes, changing the price changes the number of tickets sold. The number of tickets sold can be determined by formula or by the pattern in the relationship.

12. Is the price of the tickets and the number of tickets sold a relation? Why or why not? Yes, the data in the table can be written as a set of ordered pairs and graphed on a coordinate plane.

13. Compare the three situations you investigated: **Miles and Intersections**, **Burning Calories**, and **Ticket Prices**.
    a. How are the tables the same or different?
       Answers vary—students should identify where patterns exist and don't exist
    b. How are the graphs the same or different?
       Answers vary—students should identify where patterns exist and don't exist
    c. For which situations were you able to make a prediction?
       The kilocalorie and concert ticket problems
    d. What seems to be the connection between *dependence* and the ability to make a prediction?
       It seems that if there is a strong pattern where predictions can be made, there is also dependence between two quantities.

Ticket Prices (pp. 4 of 4) KEY

14. Examine the table and scatterplot for one activity from **Burning Calories**.
    a. How many points are plotted for each kcal value?
    Just one

    b. Is there exactly one dependent value (minutes) matched with exactly one independent value (kcal)?
    Yes

15. Examine the table and scatterplot for **Ticket Prices**.
    a. How many points are plotted for each ticket price value?
    Just one

    b. Is there exactly one dependent value (tickets sold) matched with exactly one independent value (ticket price)?
    Yes

16. The relations (*number of kcal, number of minutes*) and (*ticket price, number of tickets sold*) are both special types of relations. They are both **functions.** In both functions, there are independent and dependent variables, and more importantly, there is ***exactly*** one dependent value matched with each independent value. Do you think all relations are functions? Why or why not?
    Answers will vary. Sample: No, some relations (sets of ordered pairs) will have more than one y value for some x values.

Ticket Prices (pp. 1 of 4)

Does the *number* of concert tickets sold depend on the *price* of the ticket? If one variable depends on another, can there be more than one dependent variable for each independent variable? In this exploration, you will further investigate the idea of dependence between two quantities and functional relationships.

A popular band is scheduled to play at the Starplex Amphitheater; however, the amphitheater's management and the band cannot come to an agreement about the price of the tickets.

1. The amphitheater, which seats 50,000, has seen a drop of 150 tickets sold for each dollar increase in ticket price. The number of tickets sold can be calculated using the formula

$$t = 50000 - 150p,$$

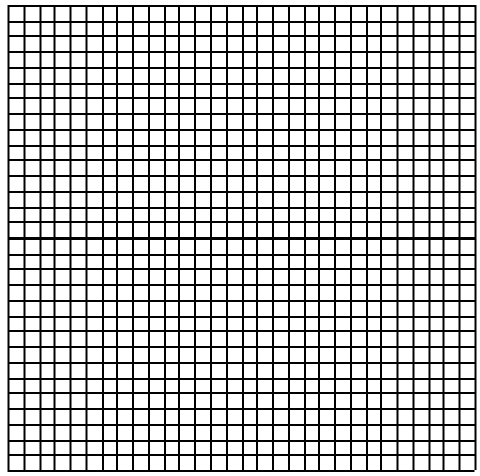
where t represents the number of tickets sold and p represents the price per ticket in dollars.

Complete the table.

Price per ticket in dollars (p)	**Process**	**Number of tickets sold (t)**
0	50000 – 150(0)	50,000
10	50000 – 150(10)	48,500
20		
30		
40		
50		
60		
p		

Ticket Prices (pp. 2 of 4)

2. Create a scatterplot. Graph the price in dollars along the *x*-axis and the number of tickets sold along the *y*-axis. Label and scale the axes appropriately.



3. How did you determine the number of tickets sold?

4. When you calculated the number of tickets sold, how many different answers did you get for each different ticket price?

5. How is your answer to question 4 reflected in the graph?

6. What are the independent and dependent variables in the problem situation?

7. Does this situation represent discrete or continuous data? Why?

8. What happens to the number of tickets sold as the price increases? How is this reflected in the table and graphs?

Ticket Prices (pp. 3 of 4)

9. Predict the price at which no tickets will be sold.

10. How did you determine the solution to question 9?

11. Does the number of tickets sold depend on the price? Why or why not?

12. Is the price of the tickets and the number of tickets sold a relation? Why or why not?

13. Compare the three situations you investigated: **Miles and Intersections**, **Burning Calories**, and **Ticket Prices**.
    a. How are the tables the same or different?

    b. How are the graphs the same or different?

    c. For which situations were you able to make a prediction?

    d. What seems to be the connection between *dependence* and the ability to make a prediction?

Ticket Prices (pp. 4 of 4)

14. Examine the table and scatterplot for one activity from **Burning Calories**.
    a. How many points are plotted for each kcal value?

    b. Is there exactly one dependent value (minutes) matched with exactly one independent value (kcal)?

15. Examine the table and scatterplot for **Ticket Prices**.
    a. How many points are plotted for each ticket price value?

    b. Is there exactly one dependent value (tickets sold) matched with exactly one independent value (ticket price)?

16. The relations (*number of kcal, number of minutes*) and (*ticket price, number of tickets sold*) are both special types of relations. They are both **functions.** In both functions, there are independent and dependent variables, and more importantly, there is ***exactly*** one dependent value matched with each independent value. Do you think all relations are functions? Why or why not?

Order of Operations (pp. 1 of 3) KEY

Part I – Order of Operations Without Calculators

To simplify a numerical expression using order of operations:

- Simplify within parentheses or grouping symbols.
- Simplify exponents (powers).
- Complete multiplication and/or division in order from left to right.
- Complete addition and/or subtraction in order from left to right.

Show all steps

1. $2 + 6 \div 2 \bullet 3$ $2 + 3 \bullet 3$ $2 + 9$ 11	2. $5(6 - 4)^3 - 5$ $5(2)^3 - 5$ $5(8) - 5$ $40 - 5$ 35
3. $36 + 4[1 + (12 - 8) \bullet 2]$ $36 + 4[1 + 4 \bullet 2]$ $36 + 4[1 + 8]$ $36 + 4[9]$ $36 + 36$ 72	4. $(7^2 - 6^2)(1^2 + 2^2)$ $(49 - 36)(1 + 4)$ $(13)(5)$ 65
5. $\dfrac{6^2 + 8^2}{4^2 - 2(7)}$ $\dfrac{36 + 64}{16 - 14}$ $\dfrac{100}{2}$ 50	6. $\dfrac{15^2 + 25^2}{2(4 + 1)}$ $\dfrac{225 + 625}{2(5)}$ $\dfrac{850}{10}$ 85

Order of Operations (pp. 2 of 3) KEY

7. Translate and then simplify the expressions.
   a. The product of nine and five, decreased by the sum of eight and seven

      $9 \bullet 5 - (8 + 7)$
      30

   b. The quotient of thirty and two, decreased by the product of two and nine

      $30/2 - 2 \bullet 9$
      -3

8. Translate and simplify the following phrase: Eighteen increased by twice the sum of three and four.

   $18 + 2(3 + 4)$
   32

9. Translate and simplify the following phrase: The difference between sixteen and one, divided by the sum of two and three.

   $(16 - 1) \div (2 + 3)$ or $\frac{16-1}{2+3} = \frac{15}{5} = 3$
   3

Order of Operations (pp. 3 of 3) KEY

<u>Practice Problems Without Calculators</u>

A. Simplify the following expressions. Show all steps.

1. $3^2 + 7^2$
58

2. $9 \bullet 2 + 3 \bullet 4$
30

3. $\frac{1}{2}(8)(12 + 14)$
104

4. $20 + 20 \div 5 + 5$
29

5. $200 - 3(5 - 1)^3$
8

6. $5 + 2\left[6 + (3 - 1) \bullet 4\right]$
33

7. $\dfrac{12^2 + 9^2 + 7^2}{4}$
68.5

8. $\dfrac{16^2 + 12^2}{5^2 - 3^2}$
25

B. Circle the simplified answer to the translation of the phrase.

9. Twelve added to the quotient of eight and four
   a. 5
   b. 11
   c. 14
   d. 1

8/4 + 12

10. Twice the sum of seven and four is divided by the product of one and eleven
   a. 1
   b. 2
   c. 3
   d. 4

$\dfrac{2(7 + 4)}{1.11}$

11. What is the first operation performed in the simplification of $4 - (3 \bullet 9 + 4)^2$?
   a. raise to a power
   b. addition
   c. multiplication
   d. subtraction

Order of Operations (pp. 1 of 3)

Part I – Order of Operations Without Calculator

To simplify a numerical expression using order of operations:

- Simplify within parentheses or grouping symbols.
- Simplify exponents (powers).
- Complete multiplication and/or division in order from left to right.
- Complete addition and/or subtraction in order from left to right.

Show all steps

1. $2 + 6 \div 2 \bullet 3$	2. $5(6 - 4)^3 - 5$
3. $36 + 4[1 + (12 - 8) \bullet 2]$	4. $(7^2 - 6^2)(1^2 + 2^2)$
5. $\dfrac{6^2 + 8^2}{4^2 - 2(7)}$	6. $\dfrac{15^2 + 25^2}{2(4 + 1)}$

Order of Operations (pp. 2 of 3)

7. Translate and then simplify the expressions.
   a. The product of nine and five decreased by the sum of eight and seven

   b. The quotient of thirty and two, decreased by the product of two and nine

8. Translate and simplify the following phrase: Eighteen increased by twice the sum of three and four.

9. Translate and simplify the following phrase: The difference between sixteen and one, divided by the sum of two and three.

Order of Operations (pp. 3 of 3)

Practice Problems Without Calculator

A. Simplify the following expressions. Show all steps.

1. $3^2 + 7^2$

2. $9 \bullet 2 + 3 \bullet 4$

3. $\frac{1}{2}(8)(12 + 14)$

4. $20 + 20 \div 5 + 5$

5. $200 - 3(5 - 1)^3$

6. $5 + 2\left[6 + (3 - 1) \bullet 4\right]$

7. $\frac{12^2 + 9^2 + 7^2}{4}$

8. $\frac{16^2 + 12^2}{5^2 - 3^2}$

B. Circle the simplified answer to the translation of the phrase.

9. Twelve added to the quotient of eight and four
   a. 5
   b. 11
   c. 14
   d. 1

10. Twice the sum of seven and four is divided by the product of one and eleven
   a. 1
   b. 2
   c. 3
   d. 4

11. What is the first operation performed in the simplification of $4 - (3 \bullet 9 + 4)^2$?
   a. raise to a power
   b. addition
   c. multiplication
   d. subtraction

Order of Operations by Graphing Calculator KEY

Simplify each expression using the calculator. Box your answer.

1. $-4^2 + 3(5 - 2)$

-7

2. $16 - 32 \div 4$

8

3. $14 - 16 \div 8 + 3^2 \bullet 5$

57

4. $10 - 3(5 - 2)$

1

5. $-3(7 + 4) - 18 \div 3^2$

-35

6. $\dfrac{5 \bullet 6 + 2}{12 - 4}$

4

7. $\dfrac{5 \bullet 2^2 + 2}{17 - 2 \bullet 3}$

2

8. $7 + 2 \bullet 28 - 3 \bullet 9 + 39 \div 3$

49

9. $3(48 \div 4) + 3(60 - 45) - 2 \bullet 8$

65

10. $7 \bullet 5 - 4(18 \div 6) + 2(7 - 4)$

29

11. $\dfrac{4 \bullet 2 + 32 \div 8}{6 - 2 - 1}$

4

12. $\dfrac{-5 \bullet 4 + 1 \bullet 15}{1 + 2 \bullet 3}$

–5/7

13. $63 - 7[18 - 3(15 \div 5)]$

0

14. $\dfrac{15 + 5[5 + 3(8 \div 4 + 2)]}{7 - 45 \div [5 + 2(6 \div 3)]}$

50

Order of Operations by Graphing Calculator

Simplify each expression using the graphing calculator. Box your answer.

1. $-4^2 + 3(5 - 2)$

2. $16 - 32 \div 4$

3. $14 - 16 \div 8 + 3^2 \bullet 5$

4. $10 - 3(5 - 2)$

5. $-3(7 + 4) - 18 \div 3^2$

6. $\dfrac{5 \bullet 6 + 2}{12 - 4}$

7. $\dfrac{5 \bullet 2^2 + 2}{17 - 2 \bullet 3}$

8. $7 + 2 \bullet 28 - 3 \bullet 9 + 39 \div 3$

9. $3(48 \div 4) + 3(60 - 45) - 2 \bullet 8$

10. $7 \bullet 5 - 4(18 \div 6) + 2(7 - 4)$

11. $\dfrac{4 \bullet 2 + 32 \div 8}{6 - 2 - 1}$

12. $\dfrac{-5 \bullet 4 + 1 \bullet 15}{1 + 2 \bullet 3}$

13. $63 - 7\left[18 - 3(15 \div 5)\right]$

14. $\dfrac{15 + 5\left[5 + 3(8 \div 4 + 2)\right]}{7 - 45 \div \left[5 + 2(6 \div 3)\right]}$

Function Notation (pp. 1 of 3) KEY

Functions can be written in two formats.

y= format	f(x) format
$y = 3x + 5$	$f(x) = 3x + 5$
$y = 2x^2 + 1$	$g(x) = 2x^2 + 1$
$y = \frac{x}{3}$	$h(x) = \frac{x}{3}$
$y = 2x$	$j(x) = 2x$

The f(x) format is called function notation. Function notation has two benefits over y= format.

- Gives different functions their specific "name." In other words f(x) denotes a specific rule, and g(x) denotes a different rule.
- It can be used to designate what value to evaluate. If it is written as f(2), it means to find rule "f" and substitute in a 2.

Example

	$f(-2)$	$f\left(\frac{1}{3}\right)$	$f(0)$	$-f(5)$	$f(j)$	$f(g)$
$f(x) = 3x + 5$	-1	6	5	-20	$3j + 5$	$3g + 5$

Practice

1.	$g(-2)$	$g(-1)$	$g(0)$	$g(5)$	$g(j)$
$g(x) = 2x^2 + 1$	9	3	1	51	$2j^2 + 1$

2.	$h(0)$	$h(-12)$	$h(3)$	$h\left(\frac{1}{3}\right)$	$h(f)$
$h(x) = \frac{x}{3}$	0	-4	1	$\frac{1}{9}$	$\frac{f}{3}$

3.	$j(0.75)$	$j(-4)$	$j(0)$	$j(f)$
$j(x) = 2x$	1.5	-8	0	$2f$

4.	$m(4)$	$m(-4)$	$m\left(\frac{1}{3}\right)$	$-m(3)$
$m(x) = 7 - 3x$	-5	19	6	2

Function Notation (pp. 2 of 3) KEY

Two area distributors, Barking Lot Grooming and Tidy Paws, sell and deliver the same kind of shampoo for dogs and cats to area veterinary clinics. The functions used by each distributor to calculate the cost to the clinics are given below.

Barking Lot Grooming	Tidy Paws
y = 5x + 3	y = 3x + 21

If both dependent variables are written as "*y*", it is hard to distinguish which equation represents which distributor.

To keep track of several functions it is sometimes necessary to distinguish them with a name. This is done by putting the functions in function or f(x) notation.

Barking Lot Grooming	Tidy Paws
b(x) = 5x + 3	p(x) = 3x + 21

5. What differences do you observe in the cost functions written in f(x) notation?
   Answers will vary. Sample: The expression remains the same only the y is changed into a letter next to (x).

6. What symbols are used to represent the dependent variable?
   b(x) for Barking Lot Grooming and p(x) for Tidy Paws

7. Write an ordered pair for each distributor using the appropriate symbols. Do not use numbers.
   Barking Lot Grooming: (x, b(x)) Tidy Paws: (x, p(x))

8. Use the appropriate function notation to evaluate the cost for 6 bottles, 9 bottles, and 15 bottles of shampoo for each distributor.

Barking Lot Grooming	Tidy Paws
b(6) = $33	b(6) = $39
b(9) = $48	b(9) = $48
b(15) = $78	b(15) = $66

9. The following function notation was given for Barking Lot Grooming: b(5) = 28.

   a. What does the 5 represent? The number of bottles of shampoo

   b. What does the 28 represent? The cost of 5 bottles of shampoo

10. The following function notation was given for Tidy Paws: p(10) = 51.

    a. What does the 10 represent? The number of bottles of shampoo

    b. What does the 51 represent? The cost of 10 bottles of shampoo

Function Notation (pp. 3 of 3) KEY

Function notation can also be used to find function values by applying the graphing calculator. Instead of naming the functions with variables, functions are named using y_1, y_2, y_3, and so on. The following steps are used to find function values in the graphing calculator.

- Put function into Y=.
- Go to Home Screen and Clear.
- Go to Vars, Y-vars, Function, Y_1. You should get Y_1 on the Home Screen. Enter (2). It should give the value of the function at 2 on the Home Screen.
- 2^{nd} Entry will bring it back up so you can type over the 2 and find another value.

11. Use the graphing calculator to check your answers on the previous problems.
    Students should check and verify answers using the graphing calculator.

Function Notation (pp. 1 of 3)

Functions can be written in two formats.

y= format	**f(x) format**
y = 3x + 5	f(x) = 3x + 5
y = $2x^2$+ 1	g(x) = $2x^2$ + 1
y = $\frac{x}{3}$	h(x) = $\frac{x}{3}$
y = 2x	j(x) = 2x

The f(x) format is called function notation. Function notation has two benefits over y= format.

- Gives different functions their specific “name”. In other words f(x) denotes a specific rule, and g(x) denotes a different rule.
- It can be used to designate what value to evaluate. If it is written as f(2), it means to find rule “f” and substitute in a 2.

Example

	$f(-2)$	$f\left(\frac{1}{3}\right)$	$f(0)$	$-f(5)$	$f(j)$	$f(g)$
f(x) = 3x + 5						

Practice

1.	$g(-2)$	$g(-1)$	$g(0)$	$g(5)$	$g(j)$
g(x) = $2x^2$ + 1					

2.	$h(0)$	$h(-12)$	$h(3)$	$h\left(\frac{1}{3}\right)$	$h(f)$
h(x) = $\frac{x}{3}$					

3.	$j(0.75)$	$j(-4)$	$j(0)$	$j(f)$
j(x) = 2x				

4.	$m(4)$	$m(-4)$	$m\left(\frac{1}{3}\right)$	$-m(3)$
m(x) = 7 – 3x				

Function Notation (pp. 2 of 3)

Two area distributors, Barking Lot Grooming and Tidy Paws, sell and deliver the same kind of shampoo for dogs and cats to area veterinary clinics. The functions used by each distributor to calculate the cost to the clinics are given below.

Barking Lot Grooming	Tidy Paws
$y = 5x + 3$	$y = 3x + 21$

If both dependent variables are written as "*y*", it is hard to distinguish which equation represents which distributor.

To keep track of several functions it is sometimes necessary to distinguish them with a name. This is done by putting the functions in function or f(x) notation.

Barking Lot Grooming	Tidy Paws
$b(x) = 5x + 3$	$p(x) = 3x + 21$

5. What differences do you observe in the cost functions written in f(x) notation?

6. What symbols are used to represent the dependent variable?

7. Write an ordered pair for each distributor using the appropriate symbols. Do not use numbers.

8. Use the appropriate function notation to evaluate the cost for 6 bottles, 9 bottles, and 15 bottles of shampoo for each distributor.

9. The following function notation was given for Barking Lot Grooming: $b(5) = 28$.

   a. What does the 5 represent?

   b. What does the 28 represent?

10. The following function notation was given for Tidy Paws: $p(10) = 51$.

    a. What does the 10 represent?

    b. What does the 51 represent?

Function Notation (pp. 3 of 3)

Function notation can also be used to find function values by applying the graphing calculator. Instead of naming the functions with variables, functions are named using y_1, y_2, y_3, and so on. The following steps are used to find function values in the graphing calculator.

- Put function into Y=.
- Go to Home Screen and Clear.
- Go to Vars, Y-vars, Function, Y_1. You should get Y_1 on the Home Screen. Enter (2). It should give the value of the function at 2 on the Home Screen.
- 2nd Entry will bring it back up so you can type over the 2 and find another value.

11. Use the graphing calculator to check your answers on the previous problems.

Flying with Functions KEY

On the back of the puzzle show the work for each problem using function notation. Verify results using the graphing calculator. After working the problems connect the dots in order.

$$f(x)=x+6 \qquad g(x)=x^2-1 \qquad h(x)=3x+4$$
$$p(x)=-x \qquad q(x)=2x^2-4x+1$$

1. f(4) 10	2. g(3) 8	3. h(-1) 1
4. q(1) -1	5. p(6) -6	6. f(-9) -3
7. g(-4) 15	8. h(5) 19	9. q(-3) 31
10. p(-9) 9	11. f(-11) -5	12. g(1) 0
13. h(-4) -8	14. q(10) 161	15. p(11) -11
16. f(-8) -2	17. g(-10) 99	18. h(3) 13
19. q(6) 49	20. p(99) -99	21. f(0) 6
22. g(2) 3	23. h(0) 4	24. q(3) 7
25. p($-\frac{1}{2}$) 1/2	26. f(0.4) 6.4	27. g($\frac{1}{2}$) -3/4
28. h(-1.2) 0.4	29. q($\frac{1}{2}$) -1/2	30. p($\sqrt{6}$) -$\sqrt{6}$

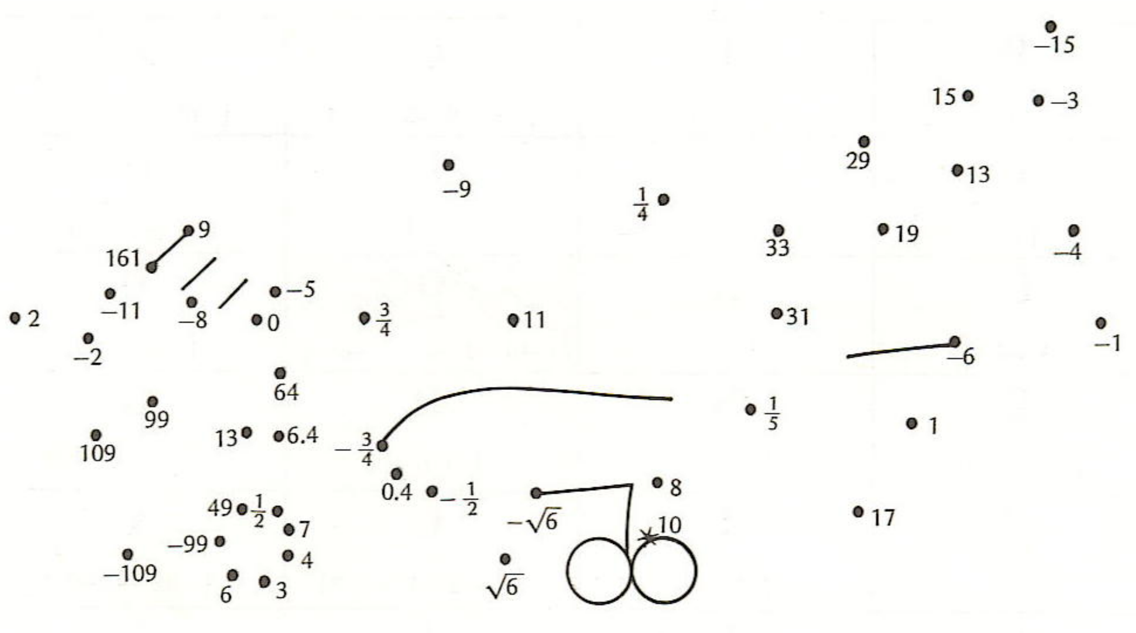
The picture is an airplane.

Flying with Functions

On the back of the puzzle show the work for each problem using function notation. Verify results using the graphing calculator. After working the problems connect the dots in order.

$$f(x) = x + 6 \qquad g(x) = x^2 - 1 \qquad h(x) = 3x + 4$$
$$p(x) = -x \qquad q(x) = 2x^2 - 4x + 1$$

1. f(4)
2. g(3)
3. h(-1)
4. q(1)
5. p(6)
6. f(-9)
7. g(-4)
8. h(5)
9. q(-3)
10. p(-9)
11. f(-11)
12. g(1)
13. h(-4)
14. q(10)
15. p(11)
16. f(-8)
17. g(-10)
18. h(3)
19. q(6)
20. p(99)
21. f(0)
22. g(2)
23. h(0)
24. q(3)
25. $p(-\frac{1}{2})$
26. f(0.4)
27. $g(\frac{1}{2})$
28. h(-1.2)
29. $q(\frac{1}{2})$
30. $p(\sqrt{6})$

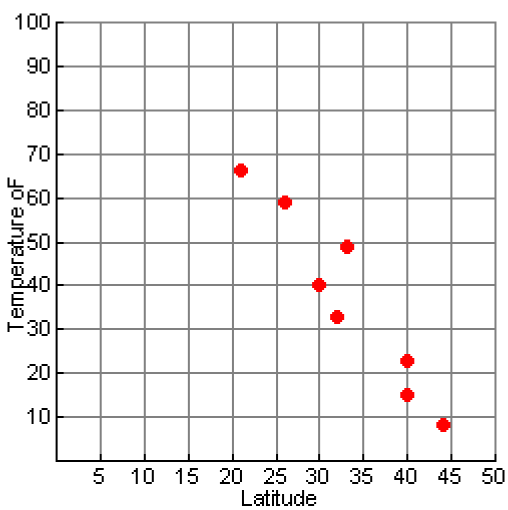


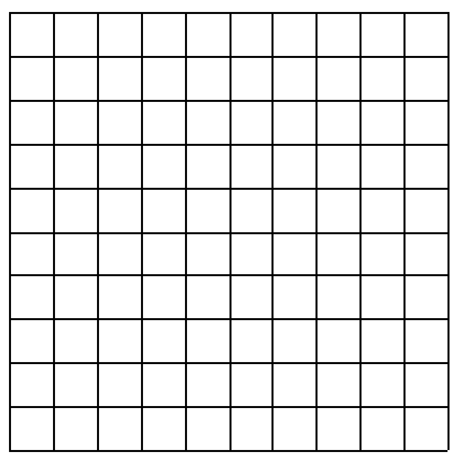
Take a Look at the Data (pp. 1 of 4) KEY

The table below shows the latitude and average daily low temperature for several cities in North America and Hawaii.

City	Latitude (°N)	Average Daily Low Temperature in January (°F)	Points (Latitude, Temp)
Miami, FL	26	59	(26, 59)
Honolulu, HI	21	66	(21, 66)
Houston, TX	30	40	(30, 40)
Philadelphia, PA	40	23	(40, 23)
Burlington, VT	44	8	(44, 8)
Jackson, MS	32	33	(32, 33)
Cheyenne, WY	40	15	(40, 15)
San Diego, CA	33	49	(33, 49)

1. Create a scatterplot of the data in the table. Graph latitude along the *x*-axis and average temperature along the *y*-axis.





2. Describe any patterns in the data.
   Answers will vary. Sample answer: As the latitude increases, the temperature decreases.

3. How do the table and scatterplot reflect the patterns in the data?
   There is no obvious pattern in the table as is. If the data were ordered it would be apparent. The scatterplot does show that as the latitude increases, the temperature decreases.

Take a Look at the Data (pp. 2 of 4) KEY

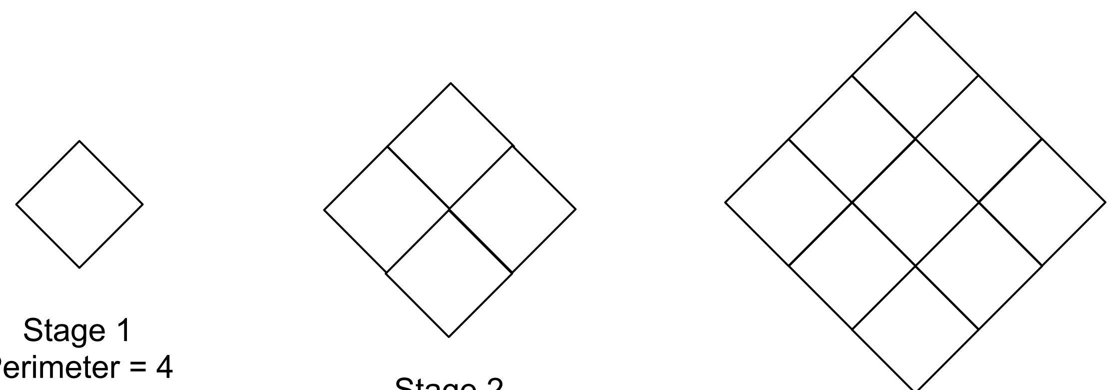
4. As the latitude increases, how does the temperature change?
   As the latitude increases, the temperature decreases.

5. Are latitude and temperature a relation? Explain your response.
   Yes, the data in the table can be written as ordered pairs.

6. What is the domain and range of the relation?
   D: {21, 26, 30, 32, 33, 40, 44}   R: {8, 15, 23, 33, 40, 49, 59, 66}

7. Does this relation represent a function?
   The relation is not a function. Cities may be at the same latitude, but they may have different temperatures, i.e., Philadelphia and Cheyenne.

Study the diagram below to determine the relationship between perimeter and stages.



Stage 1
Perimeter = 4

Stage 2
Perimeter = 8

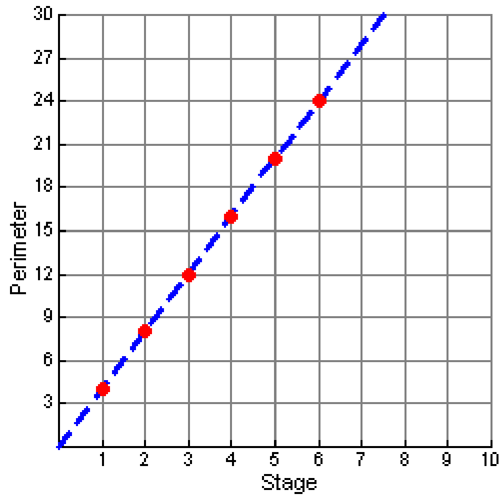
Stage 3
Perimeter = 12

8. Use the data from the diagram to fill in the table.

Stage	Process	Perimeter
1	1 • 4	4
2	2 • 4	8
3	3 • 4	12
4	4 • 4	16
5	5 • 4	20
6	6 • 4	24
x	x • 4	4x

Take a Look at the Data (pp. 3 of 4) KEY

9. Make a scatterplot of the data on the grid below. Label and scale axes over an appropriate domain and range.



10. What patterns do you observe in the diagram? How are they represented in the table and on the scatterplot?

    As x values increase, y values increase. In the table you add 4 each stage to the previous y value. On the graph as you go over one you go up four.

11. Does the data represent a relation? Explain your reasoning.

    It does represent a relation, because it is a set of ordered pairs.

12. Does the data represent a function? Explain your reasoning.

    It does represent a function, because for every x value there is a unique y value and on the graph a vertical line will only cross in one place.

13. Identify the independent and dependent variable.

    Independent – stage   Dependent – perimeter

14. Is the relationship continuous or discrete? Explain.

    The data is discrete, because the independent variable (stage) can only be a counting number.

15. Is the relationship increasing or decreasing? Explain.

    The data is increasing, because as the x value increases, the y value increases. The graph goes up from left to right.

16. Find f(34). What does this represent in the problem situation?

    $f(34) = 4(34)$ $f(34) = 136$ At Stage 34 the perimeter will be 136.

17. Is it possible in this problem situation to say that $f(x) = 85$? Explain your reasoning.

    It is not possible. Reasons will vary. Sample: No number multiplied by 4 will equal 85.

Take a Look at the Data (pp. 4 of 4) KEY

The area of a rectangular pool with a perimeter of 800 feet is given by the formula

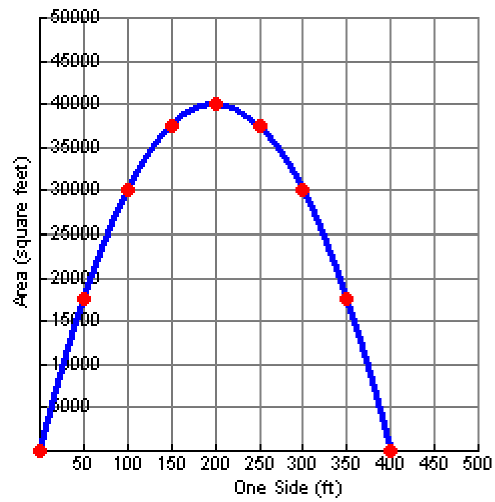
$$f(x) = 400x - x^2$$

where x represents the length of the pool in feet and f(x) represents the area of the pool in square feet.

18. Create a table.

x	f(x)
0	0
50	17500
100	30000
150	37500
200	40000
250	37500
300	30000
350	17500
400	0

19. Create a graph.



20. What patterns do you observe in the table and on the scatterplot?

    Answers will vary. Sample: The y values go up and then back down like the graph.

21. Does the relation represent a function? Explain your reasoning.

    It does represent a function, because for every x value there is a unique y value and on the graph a vertical line will only cross in one place.

22. Identify the independent and dependent variable.

    Independent – length of pool (ft)   Dependent – area of pool (ft^2)

23. Is the relationship continuous or discrete? Explain.

    The data is continuous, because the independent variable (length) can be fractional.

24. Is the relationship increasing or decreasing? Explain.

    The data is increasing up to the point (200, 40000), after which it is decreasing.

25. Find f(175). What does this represent in the problem situation?

    $f(175) = 400(175) - 175^2$ $f(175) = 39375$

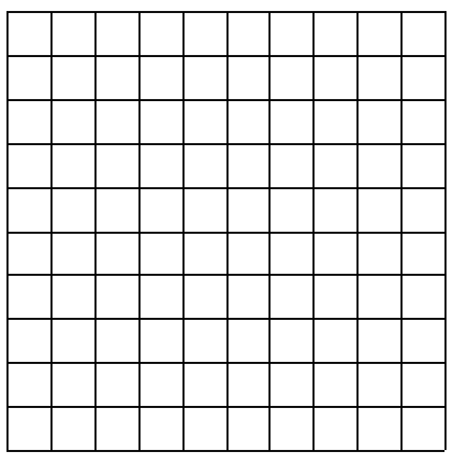
    If the length is 175 feet the area of the pool will be 39,375 ft^2.

Take a Look at the Data (pp. 1 of 4)

The table below shows the latitude and average daily low temperature for several cities in North America and Hawaii.

City	Latitude (°N)	Average Daily Low Temperature in January (°F)	Points (Latitude, Temp)
Miami, FL	26	59	
Honolulu, HI	21	66	
Houston, TX	30	40	
Philadelphia, PA	40	23	
Burlington, VT	44	8	
Jackson, MS	32	33	
Cheyenne, WY	40	15	
San Diego, CA	33	49	

1. Create a scatterplot of the data in the table. Graph latitude along the *x*-axis and average temperature along the *y*-axis.



2. Describe any patterns in the data.

3. How do the table and scatterplot reflect the patterns in the data?

Take a Look at the Data (pp. 2 of 4)

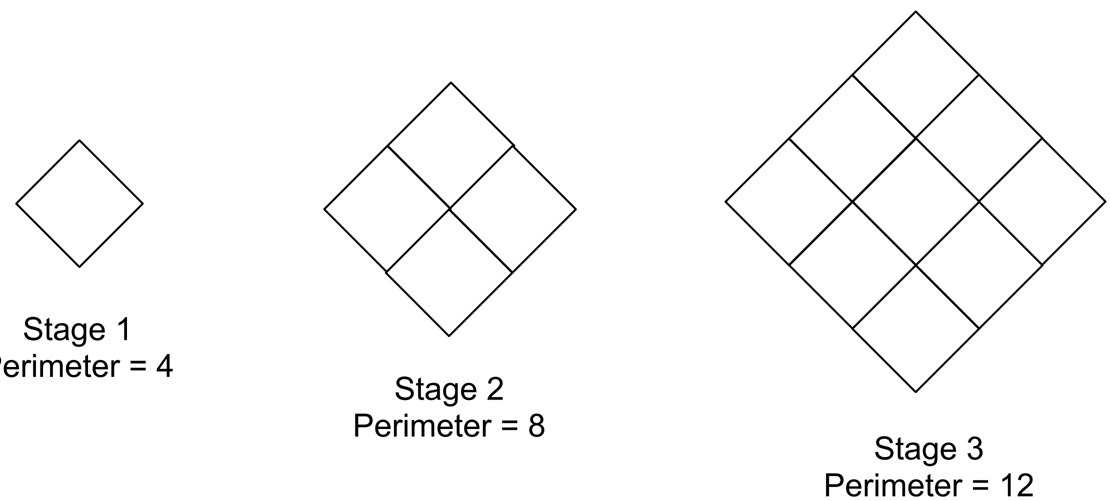
4. As the latitude increases, how does the temperature change?

5. Is latitude and temperature a relation? Explain your response.

6. What is the domain and range of the relation?

7. Does this relation represent a function?

Study the diagram below to determine the relationship between perimeter and stages.

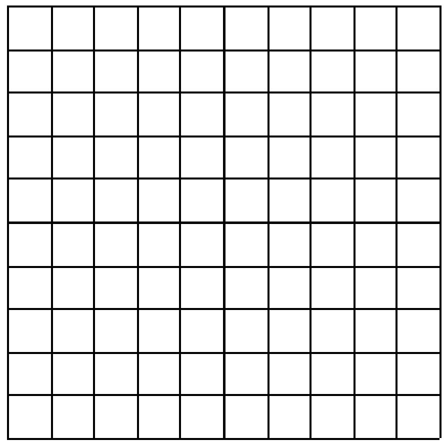


8. Use the data from the diagram to fill in the table.

Stage	Process	Perimeter
1		
2		
3		
4		
5		
6		
x		

Take a Look at the Data (pp. 3 of 4)

9. Make a scatterplot of the data on the grid below. Label and scale axes over an appropriate domain and range.



10. What patterns do you observe in the diagram? How are they represented in the table and on the scatterplot?

11. Does the data represent a relation? Explain your reasoning.

12. Does the data represent a function? Explain your reasoning.

13. Identify the independent and dependent variable.

14. Is the relationship continuous or discrete? Explain.

15. Is the relationship increasing or decreasing? Explain.

16. Find f(34). What does this represent in the problem situation?

17. Is it possible in this problem situation to say that $f(x) = 85$? Explain your reasoning.

Take a Look at the Data (pp. 4 of 4)

The area of a rectangular pool with a perimeter of 800 feet is given by the formula

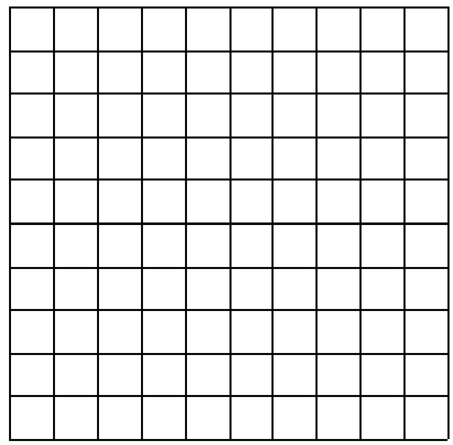
$$f(x) = 400x - x^2$$

where x represents the length of the pool in feet and f(x) represents the area of the pool in square feet.

18. Create a table.

x	f(x)
0	
50	
100	
150	
200	
250	
300	
350	
400	

19. Create a graph.



20. What patterns do you observe in the table and on the scatterplot?

21. Does the relation represent a function? Explain your reasoning.

22. Identify the independent and dependent variable.

23. Is the relationship continuous or discrete? Explain.

24. Is the relationship increasing or decreasing? Explain.

25. Find f(175). What does this represent in the problem situation?

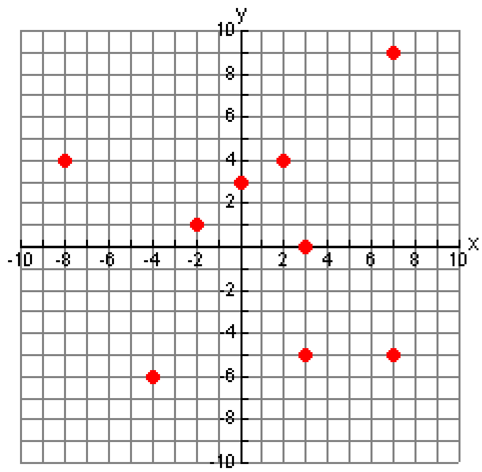
Analyzing Relations and Functions (1 of 4) KEY

1. Given the data set {(-8, 4), (7, 9), (-4, -6), (3, -5), (0, 3), (3, 0), (7, -5), (2, 4), (-2, 1)}

a. Create a table.

x	y
-8	4
7	9
-4	-6
3	-5
0	3
3	0
7	-5
2	4
-2	1

b. Create a graph.



c. What patterns, if any, do you see in the data?

No patterns are evident.

d. Is the data continuous or discrete? What are the domain and range of the data?

Discrete

D: {-8, -4, -2, 0, 2, 3, 7}   R: {-6, -5, 0, 1, 3, 4, 9}

e. Does the data represent a relation? Explain.

Yes, it is a set of ordered pairs.

f. Does the data represent a function? Explain.

No, some of the x values have more than on y value, i.e., 3 and 7

g. Could the representations of the data be used to make predictions? Explain.

No, there are no patterns to follow and no continuity in the data.

Analyzing Relations and Functions (2 of 4) KEY

2. During a treadmill test the heart rate of the patient and the amount of oxygen the patient consumes is measured. The table shows the heart rate and oxygen consumption as the treadmill's elevation was increased. The oxygen consumed can be calculated using the formula

$$c(x) = 0.014x - 0.47$$

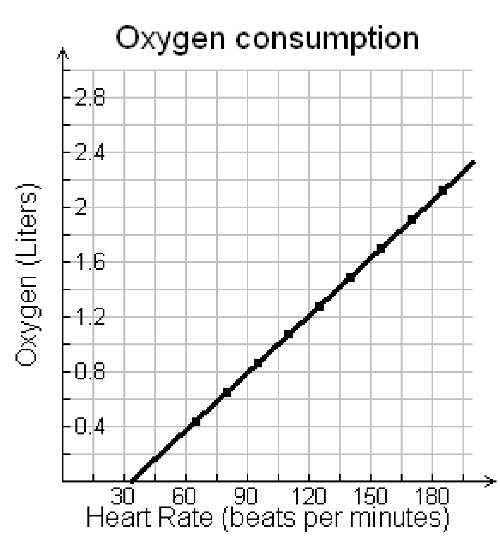
where c(x) represents the oxygen consumed and x represents the heart rate.

a. Complete the table.

Heart Rate Beats per Minute	Oxygen Consumption Liters per Minute
65	0.44
80	0.65
95	0.86
110	1.07
125	1.28
140	1.49
155	1.70
170	1.91
185	2.12

b. Which is the independent quantity? Which is the dependent quantity?
Independent – heart rate Dependent – oxygen consumption

c. Create a scatterplot of the data. Graph the heart rate along the x-axis and the oxygen consumption along the y-axis. Draw a smooth line through the scatterplot since in the real world situation partial beats can be read.
Sample graph



Analyzing Relations and Functions (3 of 4) KEY

d. Does the data represent a relation? Explain.
Yes, the data is a set of ordered pairs.

e. Does the data represent a function? Explain.
Yes, for every x value there is a unique y value and on the graph a vertical line will only cross in one place.

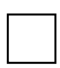
f. Is the data continuous or discrete? Explain.
The data is continuous because the heart rate can be in fractional and decimal amounts.

g. Is the data increasing or decreasing?
The data is increasing, because as the x value increases, the y value increases. The graph goes up from left to right.

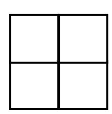
h. How can the function be described verbally?
Answers will vary. Sample: The data is linear and increasing at a constant rate.

i. Find the value of c(210). What does this represent in the problem situation?
c(210) = .014(210) - .47 = 2.47 This can also be found by extending the table or reading the graph.
With a heart rate of 210 beats per minute you consume 2.47 liters of oxygen per minute.

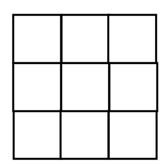
3. Study the diagram below of one-inch square tiles that are being used to determine the relationship between side length and area.



Side length = 1 in.
Area = 1 in.^{2}



Side length = 2 in.
Area = 4 in.^{2}

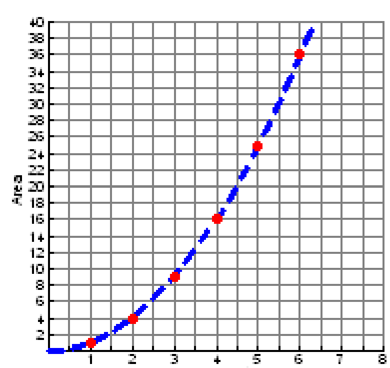


Side length = 3
Area = 9

a. Create a table.

x	Process	y
1	1^2	1
2	2^2	4
3	3^2	9
4	4^2	16
5	5^2	25
6	6^2	36
x	x^2	x^2

b. Create a graph.



Analyzing Relations and Functions (4 of 4) KEY

c. Does the data represent a relation? Explain.
Yes, the data is a set of ordered pairs.

d. Does the data represent a function? Explain.
Yes, for every x value there is a unique y value and on the graph a vertical line will only cross in one place.

e. What are the independent and dependent variables?

Independent – side length Dependent - area

f. Is the data continuous or discrete? Explain.

The data is discrete because the one-inch square tiles only occur as whole units.

g. Is the data increasing or decreasing? Explain your reasoning.

The data is increasing, because as the x value increases, the y value increases. The graph goes up from left to right.

h. How can the function be described verbally?

Answers will vary. Sample: The data is quadratic (parabolic) and increasing at a non-constant rate.

i. Find the value of f(25). What does this represent in the problem situation?

$f(25) = 25^2 = 625$ This can also be found by extending the table or reading the graph. If the sides are 25 one-inch square tiles in length, the total number of tiles, which represents the area, will be 625 $in.^2$

j. What is the value of x in $f(x) = 144$? Explain your reasoning.

The value of x is 12. Reasoning will vary. Sample: The number that squares to give 144 is 12.

k. Is $f(x) = 50$ possible in this problem situation? Explain.

It is not possible. Reasoning will vary. Sample: There is no whole number that squares to give 50. Because this involves one-inch square tiles the domain must be whole numbers.

Analyzing Relations and Functions (1 of 4)

1. Given the data set {(-8, 4), (7, 9), (-4, -6), (3, -5), (0, 3), (3, 0), (7, -5), (2, 4), (-2, 1)}

a. Create a table.

x	y

b. Create a graph.

c. What patterns, if any, do you see in the data?

d. Is the data continuous or discrete? What are the domain and range of the data?

e. Does the data represent a relation? Explain.

f. Does the data represent a function? Explain.

g. Could the representations of the data be used to make predictions? Explain.

Analyzing Relations and Functions (2 of 4)

2. During a treadmill test the heart rate of the patient and the amount of oxygen the patient consumes is measured. The table shows the heart rate and oxygen consumption as the treadmill's elevation was increased. The oxygen consumed can be calculated using the formula

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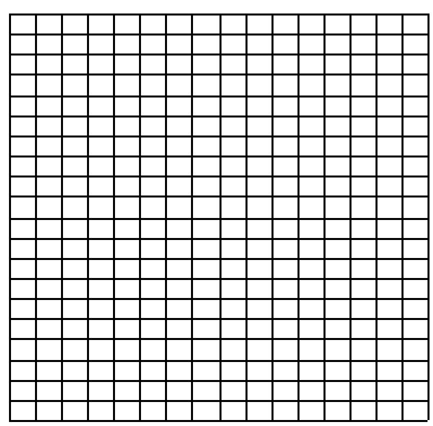
where c(x) represents the oxygen consumed and x represents the heart rate.

a. Complete the table.

Heart Rate Beats per Minute	**Oxygen Consumption Liters per Minute**
65	0.44
80	
95	
110	
125	
140	
155	
170	
185	

b. Which is the independent quantity? Which is the dependent quantity?

c. Create a scatterplot of the data. Graph the heart rate along the x-axis and the oxygen consumption along the y-axis. Draw a smooth line through the scatterplot since in the real world situation partial beats can be read.



Analyzing Relations and Functions (3 of 4)

d. Does the data represent a relation? Explain.

e. Does the data represent a function? Explain.

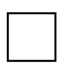
f. Is the data continuous or discrete? Explain.

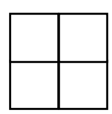
g. Is the data increasing or decreasing?

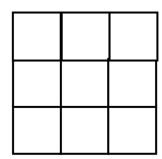
h. How can the function be described verbally?

i. Find the value of c(210). What does this represent in the problem situation?

3. Study the diagram below of one-inch square tiles that are being used to determine the relationship between side length and area.


Side length = 1 in.
Area = 1 in.^{2}

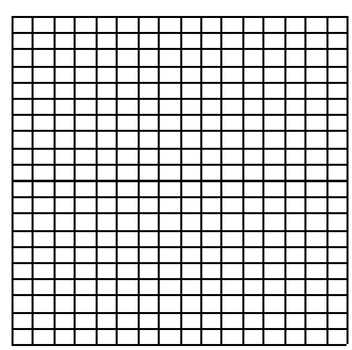

Side length = 2 in.
Area = 4 in.^{2}


Side length = _____
Area = _____

a. Create a table.

x	Process	y
1		
2		
3		
4		
5		
6		
x		

b. Create a graph.



Analyzing Relations and Functions (4 of 4)

c. Does the data represent a relation? Explain.

d. Does the data represent a function? Explain.

e. What are the independent and dependent variables?

f. Is the data continuous or discrete? Explain.

g. Is the data increasing or decreasing? Explain your reasoning.

h. How can the function be described verbally?

i. Find the value of f(25). What does this represent in the problem situation?

j. What is the value of x in f(x) = 144? Explain your reasoning.

k. Is f(x) = 50 possible in this problem situation? Explain.

